P-invariant Analysis of Timed Coloured Petri Net Models of Two Isolated Multi-Phase Traffic Light Controlled Intersections

Ganiyu R. A. (Corresponding Author)*
Olabiyisi S. O.
Omidiora E. O.
Okediran O. O.
Alo O. O.

Department of Computer Science and Engineering
Ladoke Akintola University of Technology
P.M.B 4000,Ogbomoso, Nigeria.
E-mail: ganiyurafiu@yahoo.com, Phone: +2348060596393*

ABSTRACT

In this paper, we analyze Timed Coloured Petri Net (TCPN) models of two isolated multi-phase traffic light controlled Cross-type and T-type intersections with associated fixed signal timing plans using place invariant analysis method. One of the strengths of Timed Coloured Petri Nets is the availability of the aforementioned method for analyzing a developed TCPN model in a bid to know the model's ability in allowing or disallowing occurrences of conflicting movements simultaneously. From the execution of the developed TCPN models of the Cross-type and T-type intersections, thirty and twenty-one place invariants were obtained respectively. Each of the place invariants proves that there is no possibility for any conflicting movements to have right-of-way simultaneously. Thus, the developed Timed Coloured Petri Net models of the two isolated multi-phase traffic light controlled intersections under consideration enforce the traffic operation safety rules.

Keywords: Timed Coloured Petri Net, Cross-type intersection, T-type intersection, Place invariant, Traffic light, Sub-model, Signal timing plan

1. INTRODUCTION

Owing to the growing number of vehicles, the adverse impact of urban traffic congestion cannot be overemphasized. These include excess vehicle delay, reduced safety, increased environmental pollution and fuel consumption. As a result, in urban road networks, the traffic lights at intersections regulate and guide transportation for the purpose of improving the safety and efficiency of vehicles; it manages conflicting requirements for the use of road space – often at road junctions – by allocating right of way to different sets of mutually compatible traffic movements during distinct time intervals. The control of a single intersection (belonging to the class of road traffic control) is usually based on a fixed-time strategy or a traffic-response strategy (Kutil *et al.*, 2006). In a fixed-time strategy, the light control phases (i.e. the duration of green and red light) are scheduled offline. The light control phases are derived from historical data measured in a given intersection. There are typically several light control phases for each intersection, depending on the given time of the day. It is equally important to notice that the fixed-time strategy cannot respond to any change in traffic condition since their settings are based on historical data rather than real-time data.

On the other hand, traffic-response strategy is based on feedback from the current state of traffic. It receives real-time data through sensors and creates an optimal timing plan. In a real-time control strategy, detectors located on the intersection approaches monitor traffic conditions and feed information on the actual system state to the real-time controller. Moreover, the controller selects the duration of the green phases in the signal-timing plan in order to optimize a performance index (Patel and Ranganathan, 2001; Wey, 2000; Dotoli *et al.*, 2003). In both control strategies, the traffic network has to be appropriately modelled, either for simulation purposes or in order to determine on line some states of the transportation network that are not available due to detector absence or failures (Gabard, 1991). However, in many fields of study, a phenomenon is not studied directly but through a model of the phenomenon. By the manipulation of the representation, it is hoped that new knowledge about the modelled phenomenon can be obtained without the danger, cost or inconvenience of manipulating the real phenomenon itself. Meanwhile, one way to approach the challenge of developing concurrent systems is to build an executable model of the system.

Constructing a model and simulating it usually lead to significant new insights into the design and operation of the system considered and often results in a simpler and more streamlined design. Petri nets have been proven to be a powerful modeling tool for various kinds of discrete event systems (Murata, 1989; Peterson, 1981), and its formalism provides a clear means for presenting simulation and control logic. Hence, the Petri nets are applied in traffic control. Coloured Petri net (CPN) is a graphical oriented language for modelling and validation of systems in which concurrency, communication, and synchronization play a major role. It is a discrete-event modeling language combining the classical Petri nets with the functional programming language Standard ML. More specifically, Petri nets provide the primitives for describing synchronization of concurrent processes, while programming languages provide the primitives for definition of data types and manipulation of their data values (Jensen *et al.*, 2007). Furthermore, a Coloured Petri Net model of a system is an executable model representing the states of the system as well as the events that can cause the system to change state (Jensen, 1994). The inclusion of time concepts into the Coloured Petri Net model results in a model called Timed Coloured Petri Net (TCPN) model (Ganiyu *et al.*, 2011a). In a Timed Coloured Petri Net model, a global clock models the passage of time and through this, it would be possible to calculate performance measures, such as the speed by which a system operates and throughput. Besides, TCPN models may be simulated by means of CPN Tools and verified by means of place invariant analysis.

2. REVIEW OF THE MODELS

2.1 Traffic Lights at Intersections

Traffic light signals are designed to keep traffic flowing smoothly and safely. The traffic light signals are meant to be obeyed at intersections by road users. However, the traffic lights of a generic traffic network are defined according to a signal timing plan, including green, red and yellow (amber) signals that in Nigerian cities (where available) correspond respectively to *you may go on if the way is clear*, *stop* and *stop at the stop line* (Ganiyu *et al*., 2011b).

2.2 The Case Studies

Figures 1.1 and 1.2 depict the snapshots of the two isolated multi-phase traffic light controlled intersections being modelled in this research work using Timed Coloured Petri Nets. The two intersections are of different types. That is, one is a Cross-type intersection while the other is a T-type intersection. The case studies are located in Federal Capital Territory, Abuja, Nigeria. The two intersections are controlled via traffic light signals and traffic is currently ruled in each of the intersections by a fixed time control strategy with an associated signal-timing plan. Besides, the two intersections are regularly crossed by cars, trucks, public transportation buses and mopeds. Furthermore, Figures 1.3 and 1.4 show the layouts of the Cross-type and T-type intersections respectively. Figure 1.3 consists of two roads named Festival Road and Muhammed Buhari way. This corresponds to ARTERIAL S1 and ARTERIAL S3, respectively, in Figure 1.3. With respect to the four input links of the intersection, the direction from ARTERIAL S1 is identified as North while its opposite direction is identified as South. Similarly, the direction from ARTERIAL S3 is identified as East while its opposite direction is identified as West. Similarly, Figure 1.4 consists of two roads named Herbert Macaulay Way and Ladi Kwali Street. This corresponds to ARTERIAL N5 and HOTEL COLLECTOR, respectively, in Figure 1.4. With respect to the three input links of the intersection, the direction from ARTERIAL N5 is identified as East while its opposite direction identified as West. Similarly, the direction from HOTEL COLLECTOR is identified as North.

Moreover, Table 1.1 reports the fixed signal timing plan that is currently implemented in the Cross-type intersection. In particular, the streams allowed to proceed during the phases of the signal timing plan in Table 1.1 are depicted in Figure 1.5 and labelled with letters corresponding to those indicated in Figure 1.3 and Table 1.1. More precisely, vehicle streams are represented with letters from A to H, while pedestrian streams are not included. Moreover, amber (yellow) and intergreen times are taken into account in this case, so that the considered fixed timing plan comprises 23 phases, including two amber phases (i.e., phases 10 and 11) and one intergreen time phase (i.e., phase 12). Intergreen times are short duration phases in which all traffic lights are red in a particular intersection. Indeed, no stream is allowed to proceed during such a phase. On the other hand, Table 1.2 reports the fixed signal-timing plan that is currently implemented in the T-type intersection. It consists of 15 phases. Precisely, the streams allowed to proceed during the phases of the signal timing plan in Table 1.2 are depicted in Figure 1.6 and labelled with letters corresponding to those indicated in Figure 1.4 and Table 1.2. More precisely, vehicle streams are represented with letters from A to E, while pedestrian streams are omitted.

2.3 Modelling the Cross-type and T-type Intersections

The proposed Timed Coloured Petri Net (TCPN) model of the Cross-type intersection under consideration is developed consisting of two parts, namely the *intersection sub-model* and the *traffic light sub-models*. Similarly, in order to allow easy readability, the proposed Timed Coloured Petri Net (TCPN) model of the T-type intersection under consideration is developed consisting of two parts, namely the *intersection sub-model* and the *traffic light sub-models* (Ganiyu *et. al.*, 2011b). Based on the Timed Coloured Petri Net formalism, the tokens required for both intersection and traffic light sub-models of each of the Cross-type and T-type intersections comprise three elements. The token elements (i.e. *i*, *t* and *n*) and their interpretations are as enumerated:

- The element *i* denotes the incoming vehicles into the cross-type intersection (i.e. incoming vehicles along streams C_L, D_L, G_L, H_L, M_R, N_R, O_R, P_R, A_T, B_T, E_T and F_T would to be denoted by *ic*, *id*, *ig*, *ih*, *im*, *inn*, *io*, *ip*, *ia*, *ib*, *ie* and *iff* respectively). Also, the element *i* denotes the incoming vehicles into the T-type intersection (i.e. incoming vehicles along streams A_T, B_T, C_L, D_R, E_L and H_R would to be denoted by i_a, i_b, i_c, i_d, i_e and i_h respectively).
- The element *t* represents the average time interval of vehicles entering the cross-type and T-type intersections through each stream during green phase of each cycle.
- The element *n* represents a counter counting the number of vehicles entering or leaving the cross-type and T-type intersections.

Also, the parameters necessary to describe the traffic behaviours in the Cross-type and T-type intersections are enumerated below:

- The number of vehicles entering the Cross-type and T-type intersections during green duration of each stream is measured by detectors positioned at each input link of the intersections.
- The phase durations (in seconds) of green, red and yellow signal lights of each stream as reported in signal timing plans shown in Tables 1.1 and 1.2.

2.3.1 The Intersection Sub-models of the Cross-type and T-type Intersections

The intersection sub-model requires five major places (to be drawn as circles and named *Avgt*, DATABASE, *Vr*, *Vi* and *P)*, two transitions (to be drawn as rectangular boxes and named *Movi* and *Movo*, a number of directed *arcs* connecting places and transitions, and finally some textual *inscriptions* next to the places (i.e. NI and IT) and arcs (e.g. (n, i) and (i, t)). By considering the firing sequence of the two transitions (*Movi* and *Movo*), the presence of tokens in the aforementioned places are defined as follows:

- A token in place *Avgt* represents the average time interval of vehicles entering the Cross-type and T-type intersections with respect to a particular stream.
- A token in place *Vr* means a vehicle ready to enter the Cross-type and T-type intersections.
- Place *Vi* with a token means a vehicle passing through the Cross-type and T-type intersections.
- Place *P* with a token means a vehicle ready to enter the Cross-type and T-type intersections when the green signal light turns on in the first or second instances.
- Tokens in place DATABASE represent outgoing vehicles departing the Cross-type and T-type intersections.

Meanwhile, the transition *Movi* moves an incoming vehicle into the cross-type and T-type intersections while the transition *Movo* moves an outgoing vehicle out of the Cross-type and T-type intersections.

2.3.2 The Traffic Light Sub-models of the T-type and Cross-type Intersections

To correctly control an intersection via traffic light signal indications, each traffic signal must follow a defined sequence of active colour lights, normally from green to yellow and red, and then backing to green. As a result, the traffic light parts of the proposed TCPN models modelled the changing rule of traffic lights according to the fixed signal-timing plans shown in Tables 1.1 and 1.2. In particular, vehicles are allowed to pass through each of the intersections when green lights are turned on. On the other hand, vehicles are inhibited to pass through each of the intersections during red and yellow signal indications as these, in Nigerian context, correspond to *stop* and *stop at the stop line*, respectively. By considering the Cross-type intersection shown in Figure 1.3, there are twelve vehicle streams identified, namely C-left, D-left, G-left, H-left, M-right, N-right, O-right, P-right, A-through, B-through, E-through and F-through denoted by C_L, D_L, G_L, H_L, M_R, N_R, O_R, P_R, A_T, B_T, E_T and F_T, respectively. As a result, the traffic light part of the TCPN modelling the Cross-type intersection would be divided into twelve sub-models. These are called streams C_L, D_L, G_L, H_L, M_R, N_R, O_R, P_R, A_T, B_T, E_T and F_T sub-models.

Each of the first four streams (i.e. C_L, D_L, G_L and H_L) is controlled by one set of traffic light (i.e. Red, Yellow and Green signal lights) while each of the next four streams (i.e. M_R, N_R, O_R and P_R) is controlled only by a Right turn green arrow light. However, the last four streams (i.e. A_T, B_T, E_T and F_T) are individually controlled by two sets of traffic lights. To be precise, each of the last four streams is allocated two lanes and controlled correspondingly by two sets of traffic lights placed on a long arm cantilever. On the other hand, by considering the T-type intersection shown in Figure 1.4, there are six vehicle streams identified, namely C-left, D-right, E-left, B-right, A-through and B-through denoted by C_L, D_R, E_L, H_R, A_T and B_T, respectively, in Table 1.1. As a result, the traffic light part of the TCPN modelling the T-type intersection is divided into six sub-models. These are called streams C_L, D_R, E_L, H_R, A_T and B_T sub-models. Each of the first three streams (i.e. C_L, D_R and E_L) is controlled by one set of traffic light while the next stream (i.e. H_R) is controlled only by a Right turn green arrow light. Nevertheless, the last two streams (i.e. A_T and B_T) are individually controlled by two sets of traffic lights. Precisely, each of the last two streams is allocated two lanes and controlled correspondingly by two sets of traffic lights placed on a long arm cantilever.

2.4 The Developed Timed Coloured Petri Net Models of the Cross-type and T-type Intersections

Figures 1.7 and 1.8 depict the developed Timed Coloured Petri Net models of the Cross-type and T-type intersections under consideration. However, the developed Timed Coloured Petri Net models are characterized by the following assumptions:

- One time stamp unit is assumed to represent one millisecond in each of the developed TCPN model.
- The model execution is assumed to begin when the model time (i.e. global clock) reaches one millisecond in each of the developed TCPN model.

3. ANALYZING THE DEVELOPED TCPN MODELS

A traffic light control intersection model must have certain features for proper and safe operation. Before a control strategy or model is implemented, it is necessary to make sure that the model is error free. For instance, the controller should not lock up (deadlock) due to combination of actions, should not allow conflicting movements to have right of way simultaneously. One of the strengths of Timed Coloured Petri Nets is the availability of method such as Place Invariant analysis method for analyzing a developed TCPN model. The analysis of a developed Timed Coloured Petri Net model would enlighten on the net's ability to allow or disallow conflicting movements. In analyzing each of the developed TCPN models of the Cross-type and T-type intersections under consideration, the Place Invariant analysis method is employed. It involves creating equations that are satisfied in all reachable markings by identifying sets of places over which the weighted count of tokens always remains constant.

3.1 Analysis of the Developed TCPN Model of the Cross-type Intersection

From the execution of the developed TCPN model of the Cross-type intersection shown in Figure 1.7, the following thirty P-invariants were obtained:

$$AG + AY + AR = 1 \tag{1.1}$$
$$BG + BY + BR = 1 \tag{1.2}$$
$$CG + CY + CR = 1 \tag{1.3}$$
$$DG + DY + DR = 1 \tag{1.4}$$
$$EG + EY + ER = 1 \tag{1.5}$$
$$FG + FY + FR = 1 \tag{1.6}$$
$$GG + GY + GR = 1 \tag{1.7}$$
$$HG + HY + HR = 1 \tag{1.8}$$
$$MG + P23 = 1 \tag{1.9}$$
$$NG + P24 = 1 \tag{1.10}$$
$$OG + P25 = 1 \tag{1.11}$$
$$PG + P26 = 1 \tag{1.12}$$
$$AR = BR = CR = DR = ER = FR = GR = HR = P23 = P24 = P25 = P26 = 1 \tag{1.13}$$
$$(AG + AY + AR)/(CG + CY + CR)/(PG + P26) = BR/DR/ER/FR/GR/HR/\ P23/P24/P25$$

where $BR = DR = ER = FR = GR = HR = P23 = P24 = P25 = 1$ (1.14)

$$(CG + CY + CR)/(PG + P26) = AY/BR/DR/ER/FR/GR/HR/P23/24/25$$

where $AY = BR = DR = ER = FR = GR = HR = P23 = P24 = P25 = 1$ (1.15)

$(CG + CY + CR)/(PG + P26) = AR/BR/DR/ER/FR/GR/HR/P23/P24/P25$

where $AR = BR = DR = ER = FR = GR = HR = P23 = P24 = P25 = 1$ (1.16)

$(CG + CY + CR)/(DG + DY + DR)/(PG + P26) = AR/BR/ER/FR/GR/HR/ P23/P24/P25$

where $AR = BR = ER = FR = GR = HR = P23 = P24 = P25 = 1$ (1.17)

$(CG + CY + CR)/(DG + DY + DR)/(OG + P25)/(PG + P26) = AR/BR/ER/$
$FR/GR/HR/P23/P24$

where $AR = BR = ER = FR = GR = HR = P23 = P24 = 1$ (1.18)

$(OG + P25)/(PG + P26) = AR/BR/CY/DY/ER/FR/GR/HR/P23/P24$

where $AR = BR = CY = DY = ER = FR = GR = HR = P23 = P24 = 1$ (1.19)

$(OG + P25)/(PG + P26) = AR/BR/CR/DR/ER/FR/GR/HR/P23/P24$

where $AR = BR = CR = DR = ER = FR = GR = HR = P23 = P24 = 1$ (1.20)

$(EG + EY + ER)/(FG + FY + FR) = AR/BR/CR/DR/GR/HR/P23/P24/P25/P26$

where $AR = BR = CR = DR = GR = HR = P23 = P24 = P25 = P26 = 1$ (1.21)

$(FG + FY + FR) = AR/BR/CR/DR/EY/GR/HR/P23/P24/P25/P26$

where $AR = BR = CR = DR = EY = GR = HR = P23 = P24 = P25 = P26 = 1$ (1.22)

where $AR = BR = CR = DR = ER = FR = P23 = P24 = P25 = P26 = 1$ (1.23)

$(GG + GY + GR)/(HG + HY + HR)/(MG + P23) = AR/BR/CR/DR/ER/ FR/P24/P25/P26$

where $AR = BR = CR = DR = ER = FR = P24 = P25 = P26 = 1$ (1.24)

$(GG + GY + GR)/(HG + HY + HR)/(MG + P23)/(NG + P24) = AR/BR/CR/DR/ ER/FR/P25/P26$

where $AR = BR = CR = DR = ER = FR = P25 = P26 = 1$ (1.25)

$(MG + P23)/(NG + P24) = AR/BR/CR/DR/ER/FR/GY/HY/P25/P26$

where $AR = BR = CR = DR = ER = FR = GY = HY = P25 = P26 = 1$ (1.26)

$(BG + BY + BR)/(MG + P23) = AR/CR/DR/ER/FR/GR/HR/P24/P25/P26$

where $AR = CR = DR = ER = FR = GR = HR = P24 = P25 = P26 = 1$ (1.27)

$(AG + AY + AR)/(BG + BY + BR) = CR/DR/ER/FR/GR/HR/P23/P24/P25/P26$

where $CR = DR = ER = FR = GR = HR = P23 = P24 = P25 = P26 = 1$ (1.28)

$(AG + AY + AR) = BY/CR/DR/ER/FR/GR/HR/P23/P24/P25/P26$

where $BY = CR = DR = ER = FR = GR = HR = P23 = P24) = P25 = P26 = 1$ (1.29)

$(AG + AY + AR)/(CG + CY + CR) = BR/DR/ER/FR/GR/HR/P23/P24/P25/P26$

where $BR = DR = ER = FR = GR = HR = P23 = P24 = P25 = P26 = 1$ (1.30)

The equations (1.1) – (1.30) are the P-invariants in the developed Timed Coloured Petri Net model of the Cross-type intersection. As showed by the P-invariants (1.1) – (1.13), at the initial stage of the model execution, the presence of a token in each of the places AR, BR, CR, DR, ER, FR, GR and HR shows that the red lights controlling the Stream A_T, B_T, C_L, D_L, E_T, F_T, G_L and H_L vehicles turn on while its green and yellow lights turn off. Also, the presence of a token in each of the places P23, P24, P25 and P26 indicates that a Right-turn green arrow light controlling each of the stream M_R, N_R, O_R and P_R vehicles is off. Also, the P-invariant (1.14) shows that if the places BR, DR, ER, FR, GR, HR, P23, P24 and P25 individually contain a token, then there is only one token in each set of the places (AG, AY and AR), (CG, CY and CR) and (PG and P26). That is, once a red light turns on in each set of traffic lights controlling the stream B_T, D_L, E_T, F_T, G_L and H_L vehicles, and a Right-turn green arrow light controlling each of the stream M_R, N_R, and O_R vehicles is off, then green, yellow or red lights turns on in each set of the traffic lights controlling the stream A_T and C_L vehicles, and a Right-turn green arrow light controlling the stream P_R vehicles is on or off. Thus, the P-invariant (1.14) proves that the stream A_T, C_L and P_R vehicles are compatible and those conflicting movements are not being allowed simultaneously. Similarly, the P-invariant (1.15) claims that if each of the places BR, DR, ER, FR, GR and HR contains a token and either place AY or AR also contains a token, then there must be a token in either place PG or P26, and any one of the places CG, CY and CR.

In other words, it means that once a red light turns on in the traffic light controlling the stream B_T, D_L, E_T, F_T, G_L and H_L vehicles, and yellow or red lights turns on in the traffic light controlling the stream A_T vehicles, then the traffic lights controlling the stream C_L, turn green, yellow or red while a Right-turn green arrow light controlling the stream P_R vehicles turns on or off. Hence, it is evident in P-invariant (1.15) that when the stream C_L and P_R vehicles are crossing the Cross-type intersection, all possible conflicting movements are disallowed. Conclusively, similar to the P-invariants (1.1) – (1.15), examination of the P-invariants (1.16) – (1.30) also reveals that there is no possibility for any conflicting movements to occur simultaneously in the developed TCPN model of the Cross-type intersection. Thus, the developed TCPN model enforces the traffic operation safety rules.

3.2 Analysis of the Developed TCPN Model of the T-type Intersection

From the execution of developed TCPN model of the T-type intersection depicted in Figure 1.8, the following twenty one P-invariants were obtained:

$$AG + AY + AR = 1 \tag{2.1}$$

$$BG + BY + BR = 1 \tag{2.2}$$

$$CG + CY + CR = 1 \tag{2.3}$$

$$DG + DY + DR = 1 \tag{2.4}$$

$$EG + EY + ER = 1 \tag{2.5}$$

$$HG + P6 = 1 \tag{2.6}$$

$$AR = BR = CR = DR = ER = P6 = 1 \tag{2.7}$$

$$(AG + AY + AR)/(BG + BY + BR) = CR/DR/ER/P6$$

where $CR = DR = ER = P6 = 1$ (2.8)

$$(AG + AY + AR)/(HG + P6) = (BY + BR)/CR/DR/ER$$

where $(BY + BR) = CR = DR = ER = 1$ (2.9)

$$HG + P6 = (AY + AR)/(BY + BR)/CR/DR/ER$$

where $(AY + AR) = (BY + BR) = CR = DR = ER = 1$ (2.10)

$$HG + P6 = (AY + AR)/BR/CR/DR/ER$$

where $(AY + AR) = BR = CR = DR = ER = 1$ (2.11)

$$HG + P6 = AR/BR/CR/DR/ER$$

where $AR = BR = CR = DR = ER = 1$ (2.12)

$$(EG + EY + ER)/(HG + P6) = AR/BR/CR/DR$$

where $AR = BR = CR = DR = 1$ (2.13)

$$(DG + DY + DR)/(EG + EY + ER)/(HG + P6) = AR/BR/CR$$

where $AR = BR = CR = 1$ (2.14)

$$(DG + DY + DR)/(EG + EY + ER) = AR/BR/CR/P6$$

where $AR = BR = CR = P6 = 1$ (2.15)

$$DG + DY + DR = AR/BR/CR/(EY + ER)/P6$$

where $AR = BR = CR = (EY + ER) = P6 = 1$ (2.16)

$$DG + DY + DR = AR/BR/CR/ER/P6$$

where $AR = BR = CR = ER = P6 = 1$ (2.17)

$$(AG + AY + AR)/(CG + CY + CR)/(DG + DY + DR) = BR/ER/P6$$

where $BR = ER = P6 = 1$ (2.18)

$$(AG + AY + AR)/(DG + DY + DR) = BR/(CY + CR)/ER/P6$$

where $BR = (CY + CR) = ER = P6 = 1$ (2.19)

$$AG + AY + AR = BR/CR/(DY + DR)/ER/P6$$

where $BR = CR = (DY + DR) = ER = P6 = 1$ (2.20)

$$AG + AY + AR = BR/CR/DR/ER/P6$$
where $BR = CR = DR = ER = P6 = 1$ (2.21)

The equations (2.1) – (2.21) are the P-invariants in the developed Timed Coloured Petri Net model of the T-type Intersection. As showed by the invariants (2.1) – (2.7), at the initial stage of the model execution, the presence of a token in each of the places AR, BR, CR, DR and ER shows that the red lights controlling the Stream A_T, B_T, C_L, D_R, and E_L vehicles turn on while its green and yellow lights turn off. Also, the presence of a token in place P6 indicates that a Right-turn green arrow light controlling the stream H_R vehicles is off. The P-invariant (2.8) also shows that if the places CR, DR, ER and P6 individually contain a token, then there is only one token in each set of the places (AG, AY and AR) and (BG, BY and BR). It means that once a red light turns on in each set of traffic lights controlling the stream C_L, D_R and E_L vehicles and a Right-turn green arrow light controlling the stream H_R vehicles is off, then green, yellow or red lights turns on in each set of the traffic lights controlling the stream A_T and B_T vehicles. Invariably, the invariant (2.8) depicts that the stream A_T and B_T vehicles are compatible and that they can safely cross the T-type intersection simultaneously without any accident.

In addition, the P-invariant (2.9) claims that if each of the places CR, DR and ER contains a token and either place BY or BR also contains a token, then there must be a token in either place HG or P6, and any one of the places AG, AY and AR. Obviously, it means that once a red light turns on in the traffic light controlling the stream C_L, D_R and E_L vehicles, and yellow or red lights turns on in the traffic light controlling the stream B_T vehicles, then the traffic lights controlling the stream A_T vehicles turn green, yellow or red while a Right-turn green arrow light controlling the stream H_R vehicles turns on or off. Thus, the P-invariant (2.9) establishes the fact that when the stream A_T and H_R vehicles are crossing the T-type intersection, all possible conflicting movements are disallowed. Similar to the P-invariants (2.1) – (2.9), the P-invariants (2.10) – (2.21) also verify that the rules of the control logic (such as which movements can be green at the same time in a bid to avoid any accident) related to a traffic light controlled intersection are preserved in the developed TCPN model of the T-type intersection.

4. DISCUSSION OF RESULTS

Through this medium, we have been able to analyze Timed Coloured Petri Net (TCPN) models of two isolated multi-phase traffic light controlled Cross-type and T-type intersections, with associated fixed signal timing plans, located in Federal Capital Territory, Abuja, Nigeria. The analysis of each of the developed Timed Coloured Petri Net models was carried out using Place invariant analysis method so as to know each model's ability in allowing or disallowing occurrences of conflicting movements simultaneously. From the execution of the developed TCPN model of the Cross-type intersection, thirty place invariants were obtained while twenty-one place invariants were recorded in respect of the developed TCPN model of the T-type intersection. Each place invariant of the developed TCPN model of the Cross-type intersection proves that there is no possibility for any conflicting movements to occur simultaneously. Similarly, each place invariant of the developed TCPN model of the T-type intersection also establishes the fact that the rules of the control logic (such as which movements can be green at the same time in a bid to avoid any accident) related to a T-type traffic light controlled intersection are preserved. Thus, each of the developed TCPN models enforces the traffic operation safety rules.

5. CONCLUSION AND FUTURE WORK

Using place invariant analysis method, the execution of the developed Timed Coloured Petri Net models of the Cross-type and T-type intersections yielded thirty and twenty-one place invariants respectively. Each of the place invariant gives a confirmation of the developed TCPN model's efficacy in disallowing conflicting movements to have right of way simultaneously. Thus, the verification of the developed TCPN models through place invariant analysis method would provide know-how for future research on analyzing other related multi-phase traffic light control intersections with repeated signal timing plan phases**.** Nevertheless, future and further research may be geared towards analyzing a Timed Coloured Petri Net model of a multi-phase traffic light controlled intersection, which is characterized by traffic response control strategy. Also, occurrence graph (O-graph), as one of the analysis methods of Timed Coloured Petri Nets, could be employed to verify all the dynamics properties (i.e. reachability, boundedness, home, liveness and fairness) associated with the developed Timed Coloured Petri Net models.

6. REFERENCES

Dotoli, M., Fanti, M. P. and Meloni, C. (2003). Real time optimization of traffic signal control: application to coordinated intersections. In Proc. IEEE Int. Conf. on Systems, Man and Cybernetics, Washington, 3288-3295.

Gabard, J. F. (1991). Car following models. In Concise encyclopaedia of traffic and transportation systems, pp. 65-68, M. Papageorgiou (Ed.), Oxford: Pergamon Press

Ganiyu, R. A., Omidiora, E. O., Olabiyisi, S. O., Arulogun, O. T. and Okediran, O. O. (2011a). The underlying concepts of Coloured Petri Net (CPN) and Timed Coloured Petri Net (TCPN) models through illustrative example. Accepted Manuscript for Publication in International Journal of Physical Science, Paper No: ASCN/2011/012, African University Journal Series, Accra, Ghana.

Ganiyu, R. A., Olabiyisi, S. O., Omidiora, E. O., Okediran, O. O. and Alo, O. O. (2011b). Modelling and Simulation of a Multi-Phase Traffic Light Controlled T-type Junction Using Timed Coloured Petri Nets. American Journal of Scientific and Industrial Research, 2, 3, 428-437

Jensen, K. (1994). An Introduction to the Theoretical Aspects of Coloured Petri Nets. A Decade of Concurrency, Lecture Notes in Computer Science, Springer-Verlag, 803, 230–272.

Jensen, K., Kristensen, L. M. and Wells, L. (2007). Coloured Petri Nets and CPN Tools for modelling and validation of concurrent systems. International Journal on Software Tools for Technology Transfer, 9, 3, 213–254.

Kutil, M., Hanzalek, Z. and Cervin, A. (2006). Balancing the waiting times in a simple traffic intersection model. In 11th IFAC Symposium on Control in Transportation Systems.

Murata, T. (1989). Petri Nets: Properties, Analysis and Applications. Proceedings of the IEEE, 77, 4, 541-580.

Patel, M. and Ranganathan, N. (2001). IDUTC: An Intelligent Decision-Making System for Urban Traffic-Control Applications. IEEE Trans. on Vehicular Technology, 50, 816-829.

Peterson, J.L. (1981). Petri Net Theory and the Modeling of Systems. Prentice Hall, Englewood Cliffs, New Jersey.

Wey, W. M. (2000). Model formulation and solution algorithm of traffic signal control in an urban network. Computers, environment and urban systems, 24, 355-377.

Table 1.1: Signal timing plan of the Cross-type intersection

Streams	Signal Heads	*Phases*																						
		1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*
A_T	A.1 A.2	G	Y	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	G	G	G	G	G
B_T	B.1 B.2	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	G	G	Y	R	R	R
C_L	C.1	G	G	G	G	G	Y	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	G	G
D_L	D.1	R	R	R	G	G	Y	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R
E_T	E.1 E.2	R	R	R	R	R	R	R	G	Y	Y	R	R	R	R	R	R	R	R	R	R	R	R	R
F_T	F.1 F.2	R	R	R	R	R	R	R	G	G	Y	Y	R	R	R	R	R	R	R	R	R	R	R	R
G_L	G.1	R	R	R	R	R	R	R	R	R	R	R	R	G	G	G	Y	R	R	R	R	R	R	R
H_L	H.1	R	R	R	R	R	R	R	R	R	R	R	R	G	G	G	Y	R	R	R	R	R	R	R
M_R	M.1														G	G	G	G	G					
N_R	N.I															G	G	G						
O_R	O.1					G	G	G																
P_R	P.1	G	G	G	G	G	G	G																G
Phase Duration (sec)		**15**	**3**	**2**	**4**	**3**	**3**	**3**	**6**	**1**	**2**	**1**	**2**	**2**	**1**	**9**	**3**	**2**	**1**	**12**	**3**	**3**	**8**	**1**
Cycle Duration (sec)		**90**																						

Legend G: Green signal Y: Yellow signal R: Red signal

Table 1.2: Signal timing plan of the T-type intersection

Streams	Signal Heads	*Phases*														
		1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*
A_T	A.1 A.2	G	G	Y	Y	R	R	R	R	R	R	G	G	G	G	G
B_T	B.1 B.2	G	Y	Y	R	R	R	R	R	R	R	R	R	R	R	G
C_L	C.1	R	R	R	R	R	R	R	R	R	R	G	Y	R	R	R
D_R	D.1	R	R	R	R	R	R	G	G	G	G	G	G	Y	R	R
E_L	E.1	R	R	R	R	R	G	G	G	Y	R	R	R	R	R	R
H_R	H.1		G	G	G	G	G	G								
Phase Duration(sec)		**37**	**1**	**2**	**1**	**3**	**1**	**17**	**1**	**3**	**2**	**20**	**3**	**3**	**1**	**5**
Cycle Duration(sec)		**100**														

Legend G: Green signal Y: Yellow signal R: Red signal

Figure 1.1: Snapshot of a Cross-type intersection located in FCT, Abuja, Nigeria.

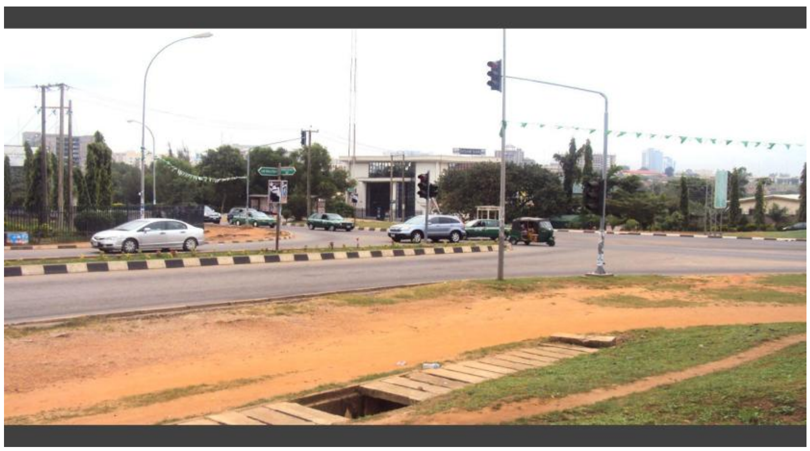

Figure 1.2: Snapshot of a T-type intersection located in FCT, Abuja, Nigeria.

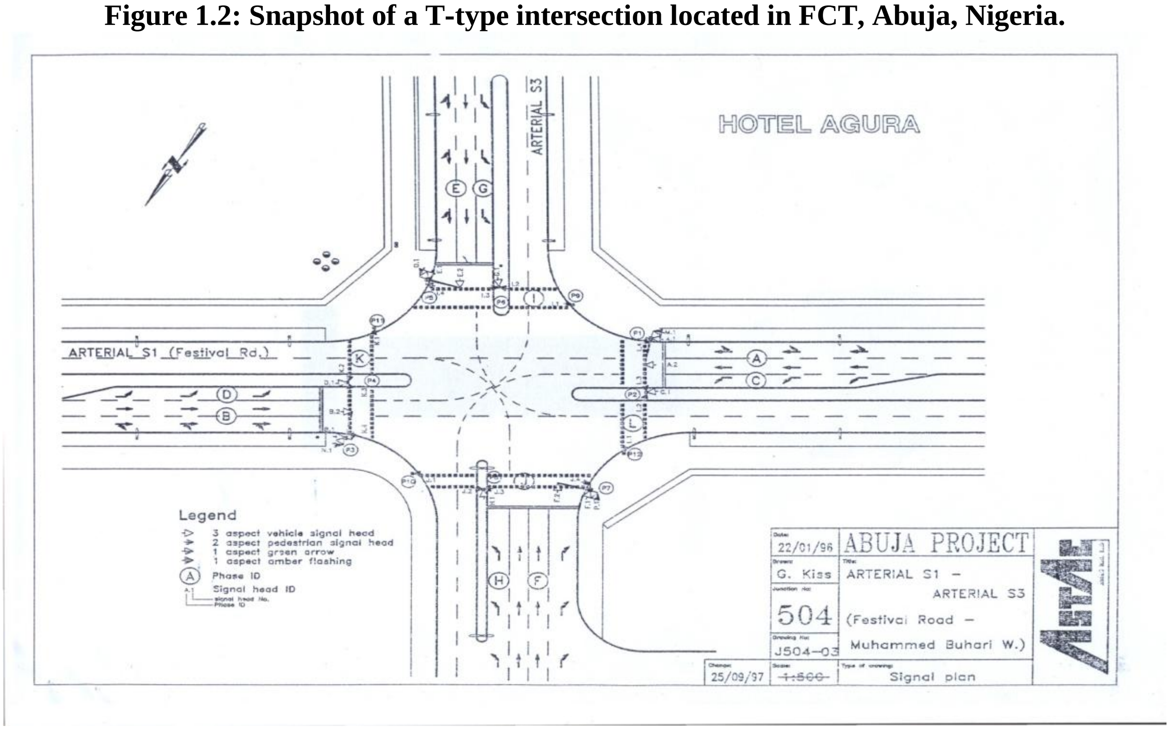

Figure 1.3: Layout of the Cross-type intersection depicted in Figure 1.1

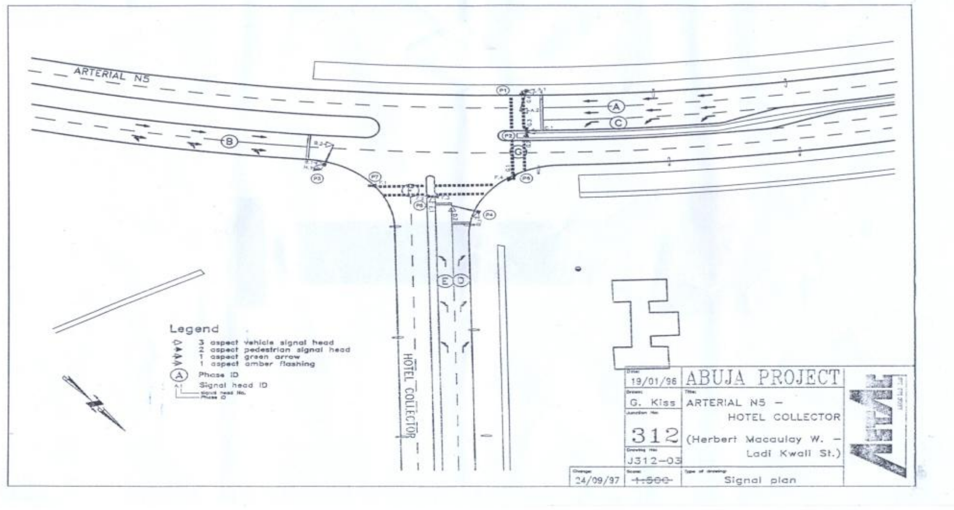

Figure 1.4: Layout of the T-type intersection depicted in Figure 1.2

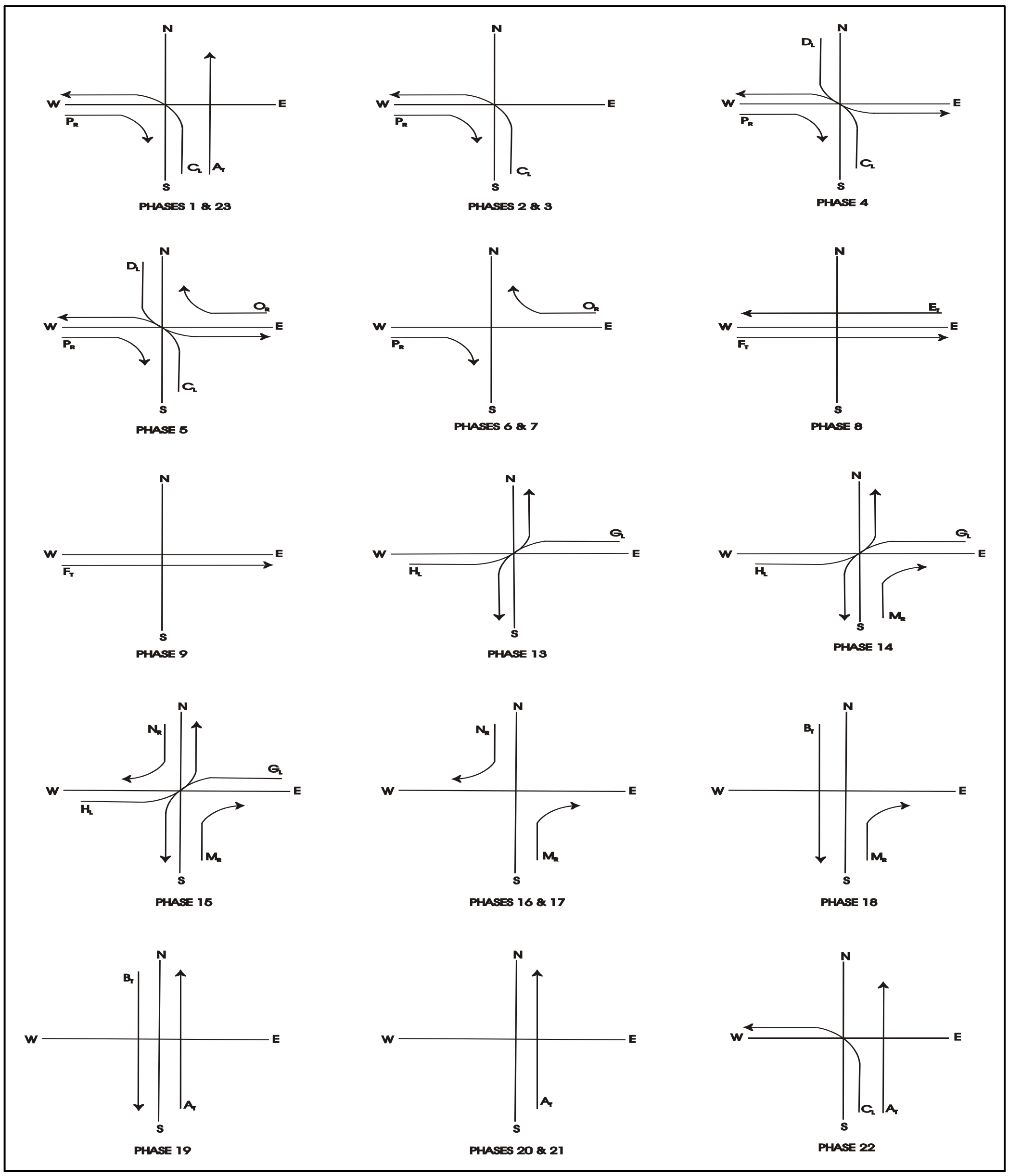

Figure 1.5: Vehicle streams in each phase of the signal timing plan of the Cross-type intersection

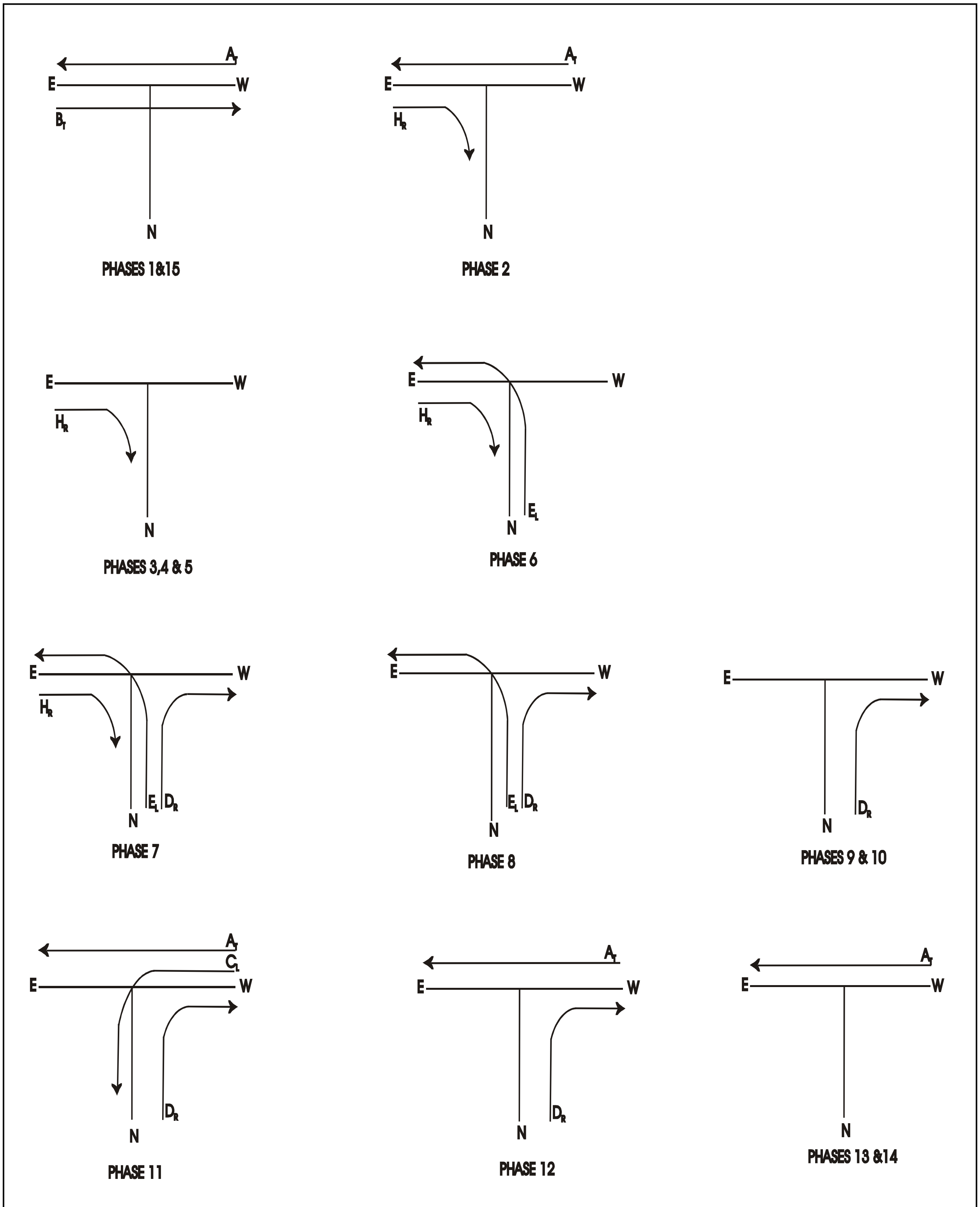

Figure 1.6: Vehicle streams in each phase of the signal timing plan of the T-type intersection

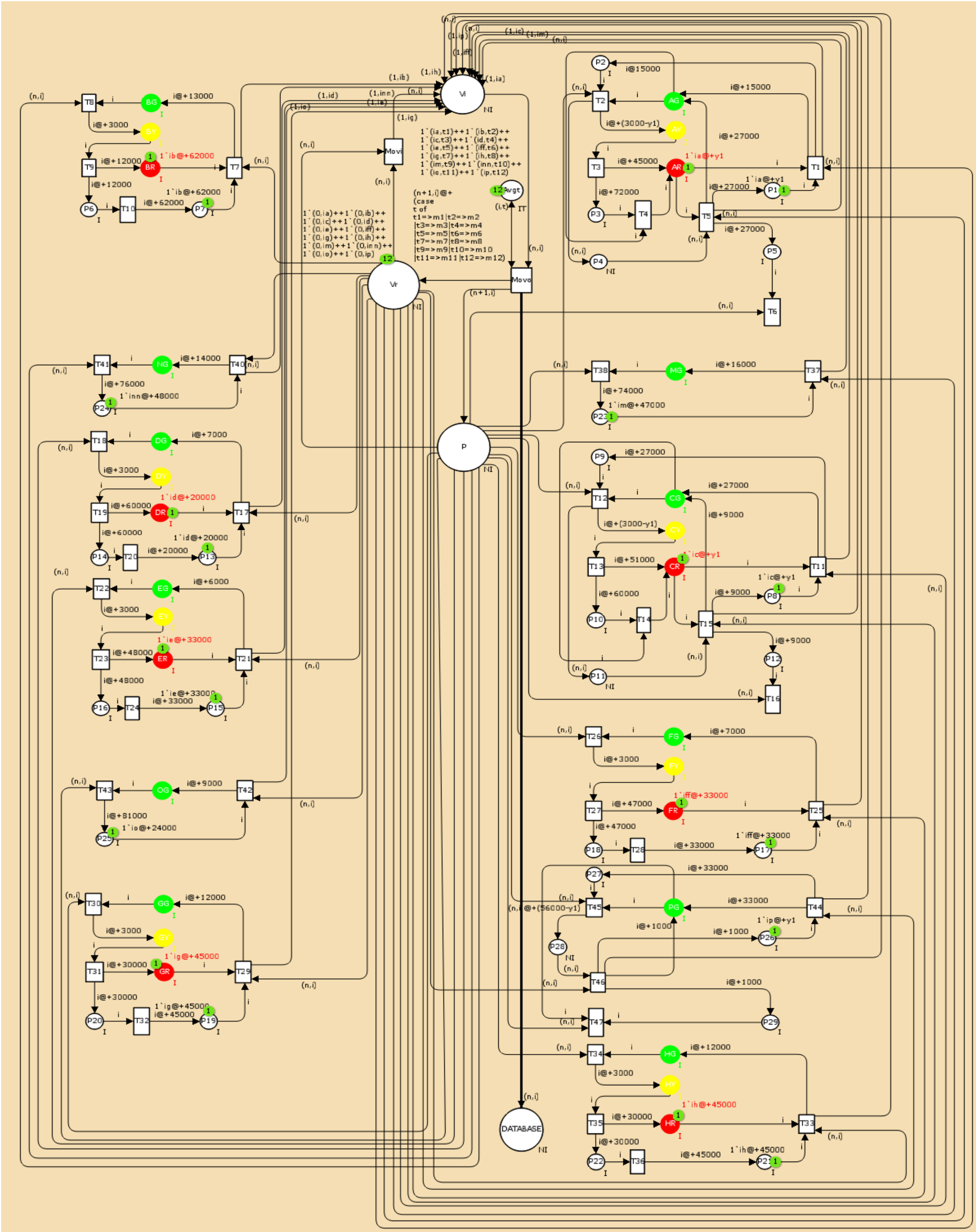

Figure 1.7: The developed TCPN model of the Cross-type intersection

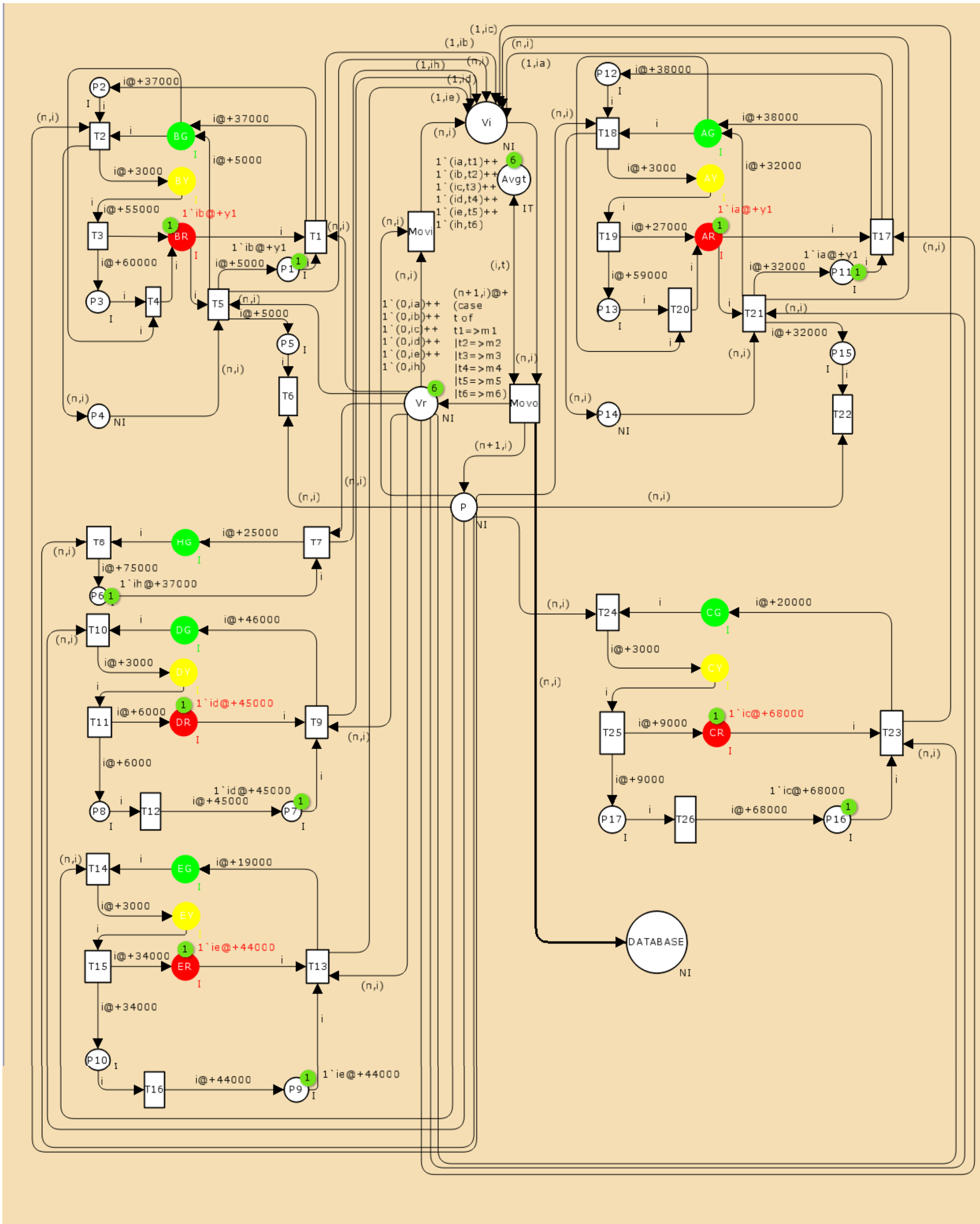

Figure 1.8: The developed TCPN model of the T-type intersection