## A MATHEMATICAL MODELING FOR DESIGN AND DEVELOPMENT OF CONTROL LAWS FOR UNMANNED AERIAL VEHICLE (UAV)

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# Abstract

Flight Mechanics and Control Laboratory of School of Aerospace Engineering -USM has been designing and developing an unmanned aerial vehicle since year 2003. The temporary result of this effort is a remotely piloted vehicle (RPV) that is controlled by radio control on the ground. In order to convert the RPV to be an unmanned aerial vehicle (UAV) that can performs the autonomous aerial surveillance and reconnaissance missions for civilian purposes, the autonomous control laws/algorithms, like damper, attitude hold/select, altitude hold/select, auto-throttle, coordinated turn and waypoints following, have been designed, and developed. This paper will discuss the step by step procedure in the design and development of the control laws of UAV starting from determination of stability/control derivatives, setting up the non-linear flight model/equation of motion, trim determination, flight dynamics analysis, designing the control laws and gain scheduling development, and the simulation of control laws in form all software simulation, the hardware in the loop simulation (HILS) and the Iron-bird simulation before doing the flight testing.

# Introduction

The UAV is a low attitude and short range UAV having following the technical specifications :

- Cruise Speed : 100 km/h
- Cruise Altitude: 1000 m
- Endurance: 2 3 Hours
- Take off weight : 20 kg, payload (camera): 5 kg
- Stall Speed : 40 km/h. and its airframe configuration is given by the Figure 1.



Figure 1: Airframe configuration of UAV

The design and development process of autonomous control laws (control algorithms) for this UAV begins with the definition of mission to be fulfilled by the UAV, which imposes requirements upon the shape of the flight path and the velocity along this flight path.

The mission requirements for the UAV are formulated as follows:

• UAV should have autonomous flight capability for aerial surveillance & reconnaissance in civil area within the defined flight envelope from the altitude 100 m to 1000 m at the speed of 75 km/h to 150 km/h.

• UAV should fly through any flight coordinates/way points precisely with good flight characteristics/ flight handling qualities.

The consequence of the requirement stated above is UAV should have following autonomous control laws/ algorithms (control modes):

- Pitch and Yaw Damper –mode to augment the stability/damping characteristics
- Attitude hold/select-mode to keep and select the desired attitude and improve the response/dynamics characteristics of UAV
- Altitude hold/select mode to maintain the desired altitude & to fly through the different altitude level
- Speed Hold mode to keep the given speed of UAV
- Coordinated Turn –mode to perform smoothly turning flight and maintain the altitude during turn flight.
- Waypoints based Auto Navigation (waypoints following)

The resulting control problem in producing the autonomous control modes is therefore to generate appropriate deflections of aerodynamic control surfaces or changes in engine power or thrust, necessary to fulfill the mission of UAV. The approach to solve this control problem is summarized in Figure 2. It illustrates a complete design & development process of autonomous control laws/algorithms for UAV and the division in different design stages starting from stability/control derivative determination, setting up the non-linear equation of motion (simulation flight model), trim determination, flight dynamics analysis, designing the control laws, gain scheduling development, until the simulation of control laws.

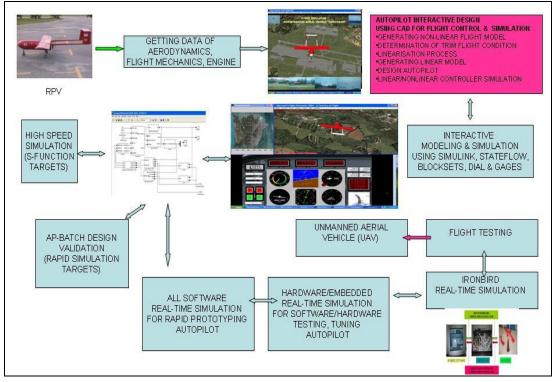


Figure 2: Procedure in design and development of control laws for UAV

## **Design and Development of Control Laws for UAV**

According to Figure 2, the process starts with calculating/ estimating data of UAV needed for non-linear flight model. It consists of the aerodynamic data, the stability and control derivatives, the engine parameter as well as the geometrical data of aircraft, like the moment inertia, the mass, the wingspan, and the wing surface.

The aerodynamic data from wind-tunnel test are compared with the calculated data. The both data closely match to each others. Figure 3 shows the final data for UAV that have been estimated using the USAF – Stability and Control DATCOM and the semi empiric formulas from ROSKAM book. The data are expressed in the body fixed coordinate system that normally is used in the flight modeling and simulation.

%		Aero	dynamic 1	Dei	rivatives			
		-0.08288;	CZO	=	-0.42155;			0.000126;
CXa	=	-0.20418;	CZa	=	-4.9323;	Cma	=	-0.4989;
CXa2	=	0;	CZa3	=	0;	Cma2	=	0;
CXa3	=	0;	CZq	=	-4.6742;	Cmq	=	-21.0042;
CXq	=	-0.03741;	CZde	=	-0.06638;	Cmde	=	-0.2358;
CXdr	=	0;	CZdeb2	=	0;	Cmb2		
CXdf	=	0;	CZdf	=	0;	Cmr	=	0;
Cladf	=	0;	CZadf	=	0;	Cmdf	=	0;
CYO		0;	C10	=	0;	Cn0	=	0;
СУЪ	=	-0.5611;	Clb	=	-0.0588;	Cnb	=	0.1488;
CYp	=	-0.0761;	Clp	=	-0.497;	Cnp	=	0.0475;
CYr	=	0.3434;	Clr	=	0.1605;	Cnr	=	-0.1654;
CYda	=	0;	Clda	=	0.3075;			-0.0276;
CYdr	=	0.0142;	Cldr	=	0.0014;	Cndr	=	-0.0064;
CYdra	=	0;	Cldaa	=	0;	Cnq	=	0;
CYbdo	t=	0;				Cnb3	=	0;
		En	gine Deri	va	tives			
		0.0132;						
Cladpt2	=	0;						
CZdpt	=	-0.0177;						

## Figure 3: Flight Data of UAV

The aerodynamic data, the stability and control derivatives as well as the engine derivatives are used as the parameter for the equations of motion describing/ modeling the motion of the UAV in the air (UAV flight model). The resulting general Earth-flat equations of motion for UAV in the body fixed coordinate system are:

$$\dot{u} = \frac{X}{m} - g\sin\theta - qw + rv$$

$$\dot{v} = \frac{Y}{m} + g\sin\Phi\cos\theta + pw + ru$$

$$\dot{\mu} = P_{pq} \cdot pq + P_{qr} \cdot qr + P_n \cdot N + P_l \cdot L$$

$$\dot{q} = Q_{pp} \cdot p^2 + Q_{pr} \cdot pr + Q_{rr} \cdot r^2 + Q_m \cdot M \dot{r} = R_{pq} \cdot pq + R_{qr} \cdot qr + R_n \cdot N + R_l \cdot L$$
(1)

These six degree of freedom (6-DOF), non-linear equations of motion describe three translating motion (force equation) and three rotating motion (moment equation) of the UAV and can cover all flight conditions and flight maneuvers in the complete flight envelope, from the take-off until landing.

To the equations of motion the kinematics equations and navigation equation below should be added. The following figure shows the graphical non-linear flight model for RPV.

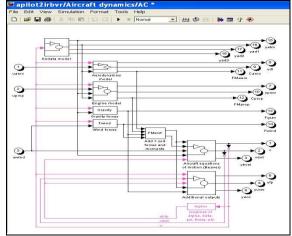


Figure 4: Simulink flight model of UAV

Once non-linear UAV flight model has been created, the next step is to determine *so* called steady-state, trim flight conditions since these conditions are a prerequisite for linearizing the non-linear model as well as for non-linear simulation. The trim flight condition is a condition in which the sums of forces and moments acting on the

aircraft are equal zero. That means that rotational and translation acceleration  $\dot{V}_T, \dot{\alpha}, \dot{\beta}, \dot{P}, \dot{Q}, \dot{R}$  in equations of motion must be equal zero.

Since the equations of motion are non-linear and the dependence of the aerodynamic data is complex, the calculation of trim flight condition is performed with numerical trim algorithm using optimization method SIMPLEX. This trim algorithm will solve for required flight variables, control surfaces and throttle setting for a desired steady-state flight condition such as a given altitude and airspeed .

The non-linear state flight model of UAV about a determined trim flight condition is linearised by computing partial derivatives of dx/dt = f(x,u) to generate the A and B matrices of linear state mode of the aircraft:

$$dx/dt = Ax + Bu$$
 (2)

where x and u now represent small deviations of state variable and control input from the trimmed steady-state values.

The partial derivatives of the output vector y = g(x,u) is taken to build the C and D matrices:

$$y = Cx + Du \tag{3}$$

where *y*, x and u are small deviations from the trim. The output variable *y* is critical variable such as accelerations and very important for controlling the aircraft motion. The JACOBIAN method is employed for calculating all derivatives of input, output and state vectors .

Finally, the linear state model matrices A, B, C, D are stored in a format suitable for the analysis software like MATLAB, see figure 5.

Based on the linear UAV model, the dynamic characteristics of UAV is analyzed, such as the trim, stability and control characteristics of the aircraft, the dynamic response of the aircraft to control input and external disturbance, the effect on the flight condition changes of the aircraft dynamics. The Analysis will be performed based on the linear UAV model as well as non-linear ones using flight simulation on the computer. After understanding the dynamic behaviors of the aircraft, the flight control laws for UAV are designed using the root locus technique regarding the good flying handling qualities given by Military flying quality requirements like MIL - STD 1797, MIL-F-8785 B.

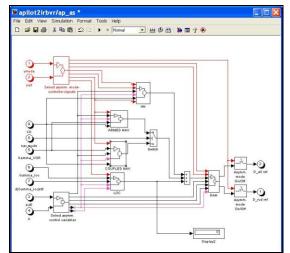
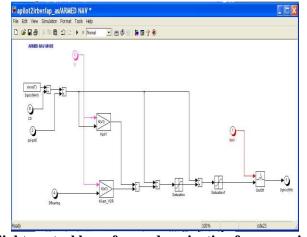


Figure 6: Modes of flight control law for the unsymmetrical flight of UAV

The UAV is designed to fly within the flight envelope from the altitude 100 m to 1000 m at the speed of 75 km/h to 150 km/h, whose boundaries are determined by angle of attack -limit, service ceiling, engine limit and airspeed limit. The UAV dynamic will changes when the UAV TAMINGSARI is flying from one flight condition to the other flight condition within this flight envelope. This can cause that a dynamic mode being stable and adequately damped in one flight condition becomes inadequately damped in other flight condition. This lightly damped oscillatory mode causes the difficulties to control UAV precisely.

This problem has been overcome by using feedback control to modify the UAV dynamics. The gain of this feedback must be adjusted according to the flight condition. The adjustment process is called *gain scheduling technique*. Here, the gains are designed for a large set of trim flight conditions and then are scheduled by interpolating them with respect to flight conditions: the gains are programmed as functions of dynamic pressure, see Figure 7 and Figure 8.





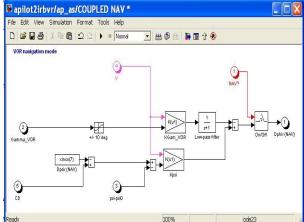


Figure 8: Flight control law of coupled navigation for waypoints following

The detailed non-linear simulation of flight control laws for UAV is made in order to validate and enhance the results of the linear control analysis, design and development. This will ensure that the flight control laws of UAV works well over the complete range of flight-envelope for which it is designed, taking into account a suitable safety margin. This analysis covers a wide range of velocities and altitudes and all possible UAV configurations. The Figure 9 & 10 shows the nonlinear, non real time simulation of the autopilot modes for UAV



Figure 9: Nonlinear, non real time simulation of waypoints following for UAV



Figure 10: Nonlinear, non real time simulation of heading hold/select for UAV

The internal structure of this controller simulation is shown in figure 11. This simulation is known as all software simulation of UAV and is used for

- Engineering design and development of control laws

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- Pilot Training
- Flight Test planning

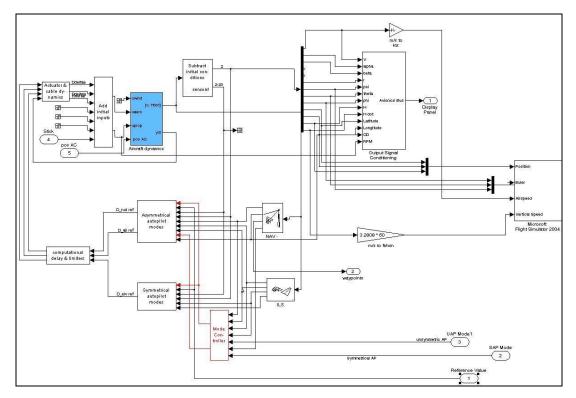


Figure 11: Internal structure of all software simulation of UAV using Simulink.

The second last step is so called hardware in-the-loop simulation or HILS. The Hardware-in-loop simulation (HILS) is a cornerstone of unmanned aircraft/UAV development. In this phase, the control laws for UAV will be evaluated in a real-time environment on the ground. Well designed simulators allow the control laws and mission functionality of UAV to be tested without risking hardware in flight test. Although HILS can not replace flight testing, it measurably reduces the likelihood of failure by detecting bugs and deficiencies in the laboratory. To facilitate this vital (and typically difficult) function, an integrated autonomous onboard computer system (real embedded controller) that is being developed will be connected to the real-time flight simulator computer to receive the measured flight variables from flight control simulator as well as to send the autonomous control surfaces signal to the flight control simulator via the external CAN interface. At the same time, the integrated avionics system will receive send data from the ground control station, as showed in Figure 12.

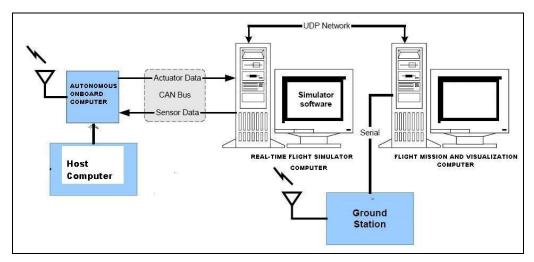


Figure 12 : Structure of HILS used in developing the UAV

The integrated autonomous onboard computer consists of a micro controller MPC 555, an air data and altitude heading reference system (ADAHRS), on-board data link, on-board GPS antenna as well as the port for servo motor, see Figure 13.

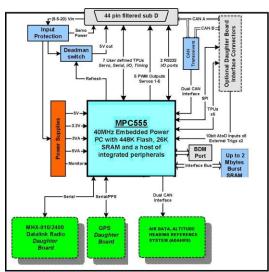


Figure 13: System architecture of integrated autonomous onboard computer

Since the design and development of control laws for UAV TAMINGSARI have used the software MATLAB and Simulink augmented with the autocode tools Real-Time-Workshop (RTW) and Stateflow, so the graphically flight model of UAV TAMINGSARI and its flight control laws can then be automatically coded in C using RTW, compiled using the software environment, and then downloaded to the UAV integrated onboard computer system (real embedded controller). This embedded controller has more than enough CPU muscle to run complicated autocoded algorithms.

The final step before the flight testing is what so called the IRONBIRD simulation. The IRONBIRD simulation is as final check for system configuration and used for measuring the closed-loop response of control laws, to verify actuator models. The Figure 14 shows the configuration of IRONBIRD simulation for UAV.

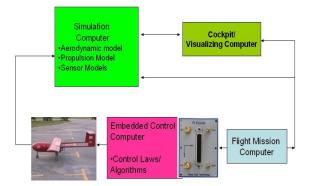


Figure 14: Ironbird simulation of UAV.

In this simulation, the autonomous onboard computer in which the flight control laws reside will be put into the UAV airframe and connected with the real servo actuators of UAV to replace the mathematical model of actuator of UAV.

#### Conclusion

The design and development of the control laws/algorithms for UAV is not only just designing and simulating the linear control laws, but there are some issues such as getting the UAV data (aerodynamics, stability & control, engine), generating the nonlinear & linear model, trim determination, the gain scheduling, non-linear simulation of control laws as well as the real time simulation of the control laws on the hardware environment (hardware in the loop and iron bird simulation). These issues have to be done and solved in order to convert the remotely piloted vehicle into fully autonomous UAV before the flight test of UAV is done. Actually these procedures/steps are common ones in designing and developing the automatic flight control system for the aircraft

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