Radiative Double Diffusive Free Convective Flow towards a Stagnation Point with Catalytic Surface Reaction

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Abstract

In this work we consider the double diffusive free convection flow of an incompressible fluid with radiation and catalytic surface reaction toward a stagnation point, the governing partial differential equations were transformed into a system of coupled ordinary differential equations using method of similarity and are solved using the fourth order runge-kuta iteration technique together with the shooting method, effects of the various thermo-physical parameters were also investigated and visualized graphically.

Keywords: Radiative, double diffusive, stagnation point, surface reaction

Introduction

Studies involving surface reaction have become important due to its varied application in both scientific and industrial applications. These reaction often proceeds slowly therefore requires the effect of catalyst to speed off these reactions[Pop and Ingham2001]. Meanwhile the study of the transport processes and there interaction with chemical reactions and thermal radiation are quite difficult and are intimately connected to the underlying hydrodynamics of the process[Makinde et'al, 2011]. More over the study of the heat generation and absorption effects in moving fluids are important in several studies involving exothermic and endothermic chemical reactions.Several studies have been done involving chemical reactions in boundary layer flow these includes both experimental and theoretical analysis such as the work of Merkin and chaudhary[1994,1996], Chaudhary et'al[1995].who proposed a model for free convective boundary layers which are generated purely by heat supplied to the surroundings fluid by exothermic surface reactions. They assumed that on the surface the catalyst the reaction is modeled by a single first order Arrhenius kinetics. They neglected the effect of the concentration induced buoyancy term and thermal radiation.

Makinde et'al[2011] also studied the unsteady convection with chemical reaction and radiative heat transfer past a flat porous plate moving through a binary fluid but neglected surface reaction and radiation, in their work they assumed a one dimensional flow and that the reactions takes place in the whole boundary layer. The aim of this study is therefore to investigate the steady boundary layer flow of an incompressible fluid with catalytic surface reaction and radiative heat transfer, using a similarity transform together with the Runge-Kuta iterative shooting technique. The analysis of the results obtained shows that the flow field is influenced by the surface reaction, thermal radiation, thermal Grasshof number, solutal Grasshof number and the Schmidt number.

Mathematical Model

Consider the convective flow of with surface reaction and radiative heat transfer of an optical thick fluid toward a stagnation point. We choose the catersian axes (x,y) parallel and perpendicular to the flow field with velocity components (u,v) respectively. Under these conditions the momentum, energy and chemical species concentration balance equations governing the problem may be written as [Pop and Ingham, 2001].

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\gamma\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)S(x)+g\beta(C-C_\infty)S(x) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2} \tag{4}$$

subject to the following boundary conditions

$$u=U_w(x), v=0, k_f\frac{\partial T}{\partial y}=-Qk_0C\exp[-\frac{E}{RT}], D\frac{\partial C}{\partial y}=k_0C\exp[-\frac{E}{RT}], as\ y=0 \tag{5}$$

$$u\to 0, T\to T_\infty, C\to C_\infty\ \ at\ x=0\ and\ y\to\infty \tag{6}$$

Where u and v are the velocity components on the x and y directions respectively, T is the temperature, g is the acceleration due to gravity, β is the coefficient of expansivity, D is the molecular diffusion coefficient, C is the mass concentration, T_∞ is the free stream temperature, C_∞ is the free stream concentration, Uw(x) is the wall velocity, Q is the heat of reaction k_0 is a constant S(x) is a body function E is the activation energy, R is the universal gas constant.

In order to solve equations (1)-(6) above we introduce the following dimensionless quantities

$$U_w(x)=U_0x,\ S(x)=x,\ \eta=y\sqrt{\frac{U_w(x)}{\gamma x}},\ \psi=\sqrt{\gamma U_w(x)x}\,f(\eta), v=-\frac{\partial\psi}{\partial x}, u=\frac{\partial\psi}{\partial y}, G_t=\frac{g\beta T_\infty}{U_0^2}$$

$$G_c=\frac{g\beta C_\infty}{U_0^2}, \varepsilon=\frac{E}{RT_\infty}, \alpha 1=\frac{1}{k_f}\sqrt{\frac{\gamma}{U_0}}Qk_0\frac{C_\infty}{T_\infty}EXP[\frac{-E}{RT_\infty}],\ \alpha 2=\frac{k_fT_\infty}{C_\infty Q},\ \Pr=\frac{\gamma}{\alpha}, Sc=\frac{\gamma}{D}, R=\frac{4k_ek}{\sigma T_\infty^3},$$

$$\theta(\eta)=\frac{T}{T_\infty}, \varphi(\eta)=\frac{C}{C_\infty}$$

Using the above dimensionless quantities and the Rosseland approximation for an optically thick gas we have the following coupled differential equations.

$$f'''+ff''-f'^2+G_t(\theta-1)+G_c(\varphi-1)=0 \tag{7}$$

$$\theta''+\frac{3\Pr}{4+R}f\theta'=0 \tag{8}$$

$$\varphi''+Scf\varphi'=0 \tag{9}$$

the transformed boundary conditions are

$$f'(0)=1, f(0)=0, \theta'(0)=-\alpha 1\varphi(0)EXP[\varepsilon(1-\frac{1}{\theta(0)}), \varphi'(0)=\alpha 1\alpha 2\varphi(0)EXP[\varepsilon(1-\frac{1}{\theta(0)}), f'(\infty)=0,$$

$$\theta(\infty)=1, \varphi(\infty)=1$$

Where prime represents derivative with respect to η, $\alpha 2$ is the reactant consumption parameter, $\alpha 1$ is the Damkohler number, Pr is the Prandtl number, Sc is the Schmidt number, G_t is the thermal Grasshof number, G_c=solutal grasshof number ε is the activation energy parameter, R is the radiation parameter. Other physical parameter of interest are the skin friction, Nusselt number and Sherwood number which are numerically determined in this study. They are defined in dimensionless terms as , $\tau(0)=-\ f''(0)$, Nu=θ', Sh=$\varphi'(0)$.

Numerical Computation

The set of coupled non-linear ordinary differential equations (7)-(9) together with the associated boundary conditions have been solved numerically by using the Runge-Kuta integration technique together with the iterative shooting procedure. The computations were carried out by a program which uses a symbolic and computational computer language MAPLE.

Results and Discussion

In order to get insight into the physical significance of the problem the velocity, temperature and concentration have been computed and visualized graphically. The values of the skin friction, Nusselt and Sherwood numbers have been computed for various values of the physical parameters. These results are shown in Table.1 and Figs.1-11. In table.1 we show the computed values of the skin friction, Nusselt and Sherwood numbers, it is seen that the skin friction deceases while the Nusselt and Sherwood numbers increases for increasing values of $\alpha 1 = 0.1, 0.15, 0.2, 0.25, 0.3$ while the skin friction and Sherwood number increases the Nusselt number decreases for increasing values of $\alpha 2 = 0.1, 0.15, 0.2$ which signifies an increase in consumption of the reactant. Its seen that increase in the thermal Grasshof number(Gt=1, 3, 5) increases the Nusselt and Sherwood numbers while they decrease for the solutal Grasshof number(Gc=10, 30, 50). While the skin friction decreases with increase in the radiation parameters(R=1, 1.2, 1.4) the Nusselt and Sherwood numbers increase slightly.

Table.1: Effects of various parameters on the skin friction, Nusselt number and Sherwood number.

$\alpha 1$	$\alpha 2$	pr	Gt	Gc	R	Sc	- $f''(0)$	- $\theta'(0)$	$\varphi'(0)$
0.1	0.1	0.71	1	1	1	1	0.72768055506254	0.09874176661391	0.00987417666139
0.15							0.60942615093937	0.14723999189051	0.01472399918905
0.2							0.49744705497368	0.19519587648321	0.01951958764832
0.25							0.39035805515496	0.24263226588703	0.02426322658870
0.3							0.28730264654179	0.28956947491624	0.02895694749162
0.35							0.18769333122968	0.33602584639790	0.03360258463979
0.1	0.1	0.71	1	1	1	1	0.72768055506254	0.09874176661391	0.00987417666139
	0.15						0.72989159391244	0.09800493951463	0.01470074092719
	0.2						0.73207489858503	0.09727810123793	0.01945562024758
0.1	0.1	0.71	1	1	1	1	0.72768055506254	0.72768055506254	0.00987417666139
		3					0.96849498590373	0.09839431840359	0.00983943184035
		7					0.99327243024372	0.09834259459334	0.00983425945933
0.1	0.1	0.71	1	1	1	1	0.72768055506254	0.09874176661391	0.00987417666139
			3				0.27072925388710	0.09889759927307	0.00988975992730
			5				0.14091890247898	0.09898493716543	0.00989849371654
0.1	0.1	0.71	1	10	1	1	1.1298156801162	0.09892563329100	0.00989256332910
				30			1.22305210944565	0.09892010750187	0.00989201075018
				50			1.31806444397247	0.09891437936922	0.00989143793692
0.1	0.1	0.71	1	1	1	1	1.08841051798459	0.09892805765320	0.00989280576532
					1.2		1.08707709008551	0.09893018534192	0.00989301853419
					1.4		1.08595912921676	0.09893195816309	0.00989319581630
0.1	0.1	0.71	1	1	1	1	1.08841051798459	0.09892805765320	0.00989280576532
						1.5	1.0877526518014	0.09904102791983	0.00990410279198
						2	1.08721233851373	0.09913511211897	0.00991351121189

The velocity profiles are depicted in Figs.1,6,8,9 and 10 below. It is seen that the velocity profiles increases for increase in the reaction rate parameter $\alpha 1$, the radiation parameter R, and the thermal Grasshof number while it decreases for the solutal Grasshof number and the prandtl number. The profile of the temperature is shown in Figs.2,7 and 11. It is observed that for the rate constant and the radiation term the temperature profile is increasing while it decreases with the prandtl number. The response of the concentration profile to the various parameter of interest is shown in Figs.3, 4 and 5. We observed that the concentration decreases with increasing values of the reaction rate and reactant absorption parameters while it increases with increase in the Schmidt number.

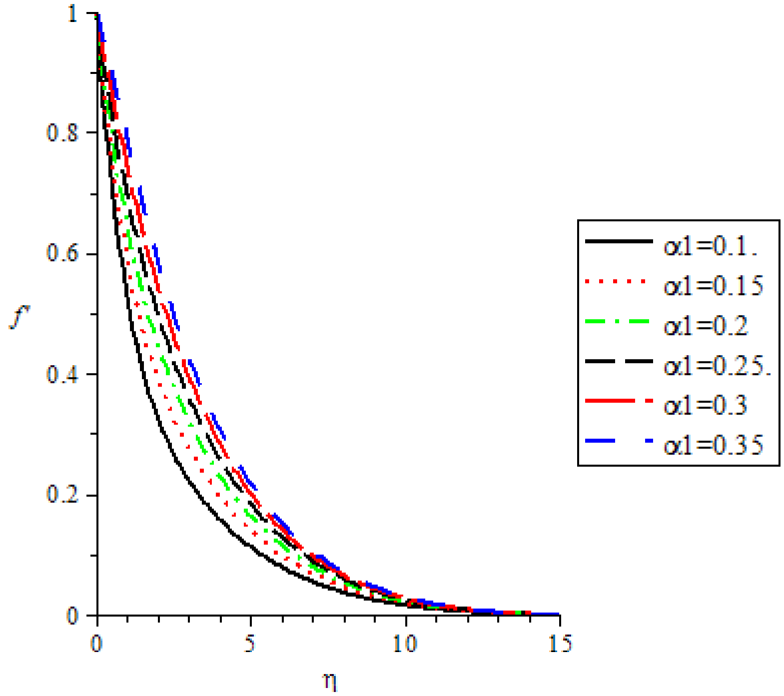

Fig.1: Velocity profile for various values of $\alpha 1$ for
$Pr = 0.71$, $Gt = 1$, $Gc = 0.1$, $R = 1$, $\epsilon = 0.01$, $\alpha 2 = 0.1$, $Sc = 1$

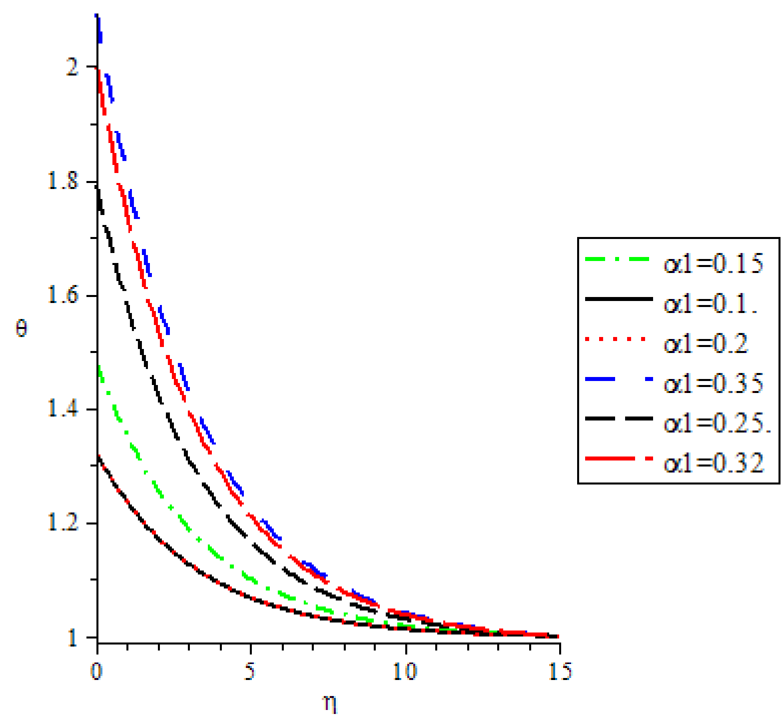

Fig.2: Temperature profile for various values of α1 for
$Pr = 0.71$, $Gt = 1$, $Gc = 0.1$, $R = 1$, $\epsilon = 0.01$, $\alpha 2 = 0.1$, $Sc = 1$

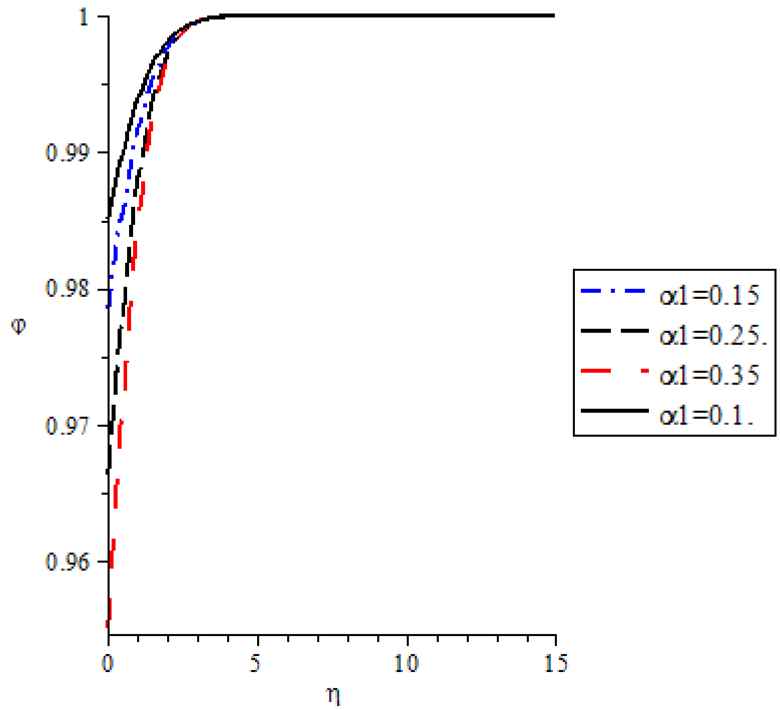

Fig.3:Concentration profile for various values of α1 for
$Pr = 0.71,\ Gt = 1,\ Gc = 0.1,\ R = 1,\ \epsilon = 0.01,\ \alpha 2 = 0.1,\ Sc = 1$

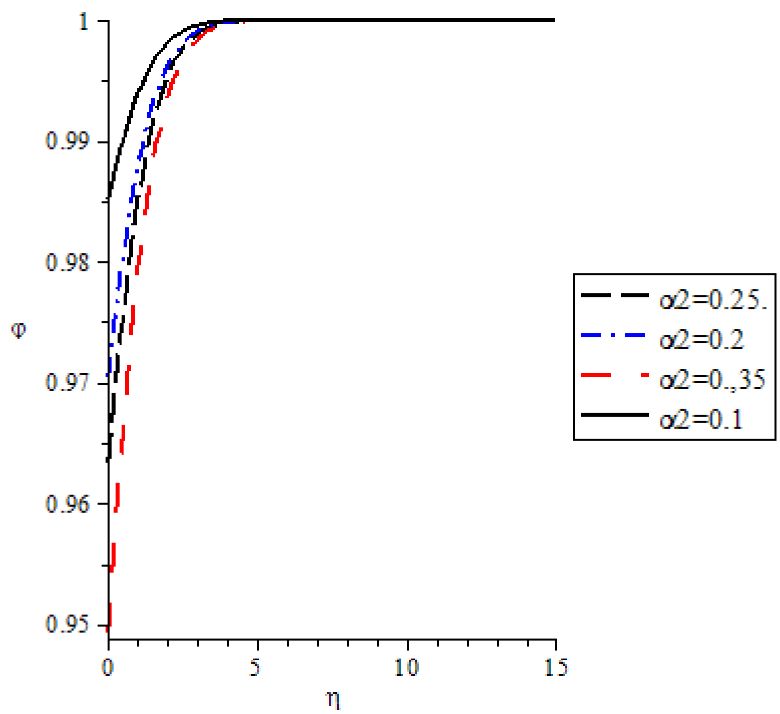

Fig.4:Concentration profile for various values of α2 for
$Pr = 0.71,\ Gt = 1,\ Gc = 0.1,\ R = 1,\ \epsilon = 0.01,\ \alpha 1 = 0.1,\ Sc = 1,$

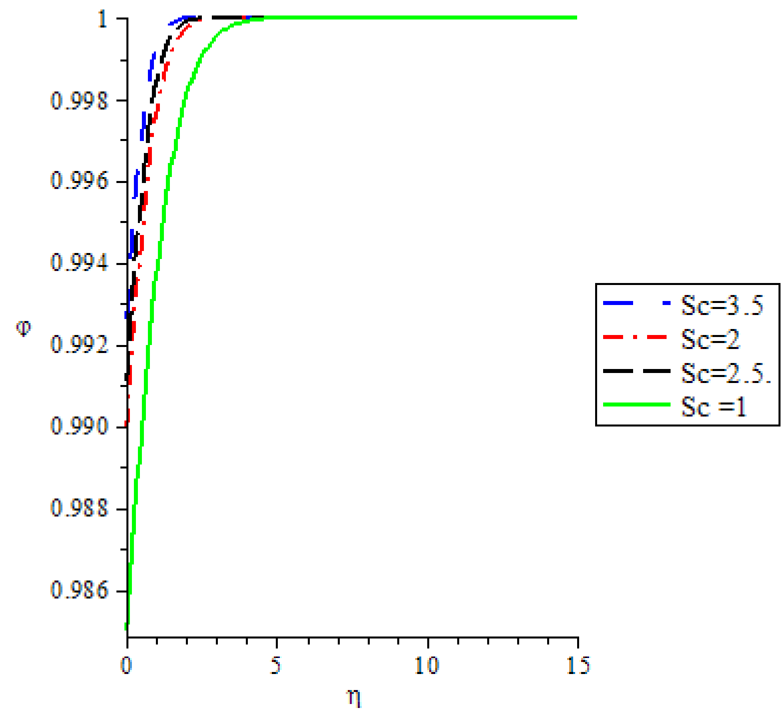

Fig.5:Concentration profile for various values of schmidt number for
$Pr = 0.71,\ Gt = 1,\ Gc = 0.1,\ R = 1,\ \epsilon = 0.01,\ \alpha 1 = 0.1,\ Sc = 1,$

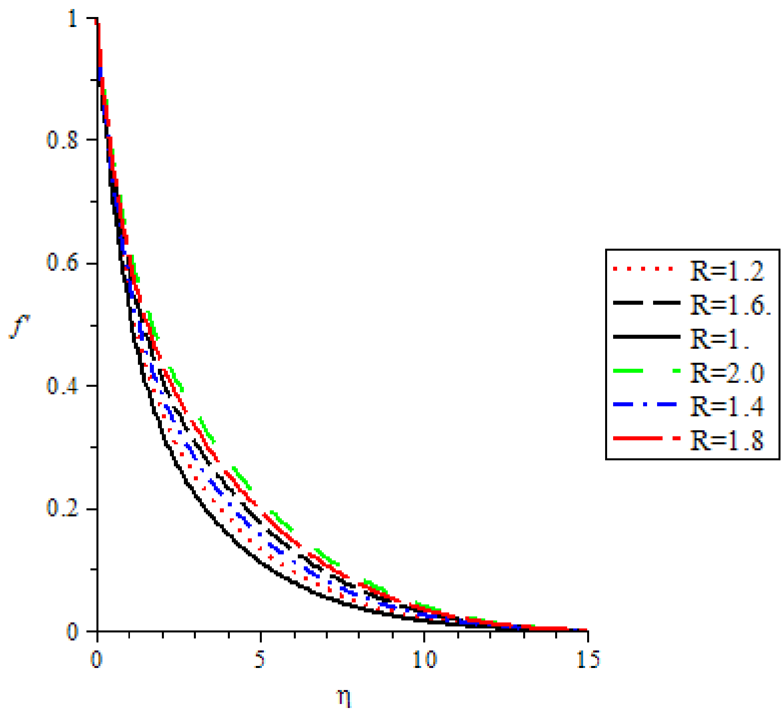

Fig.6: Velocity profile for various values of radiation number for
$Pr = 0.71,\ Gt = 1,\ Gc = 0.1,\ R = 1,\ \epsilon = 0.01,\ \alpha 2 = 0.1,\ Sc = 1$

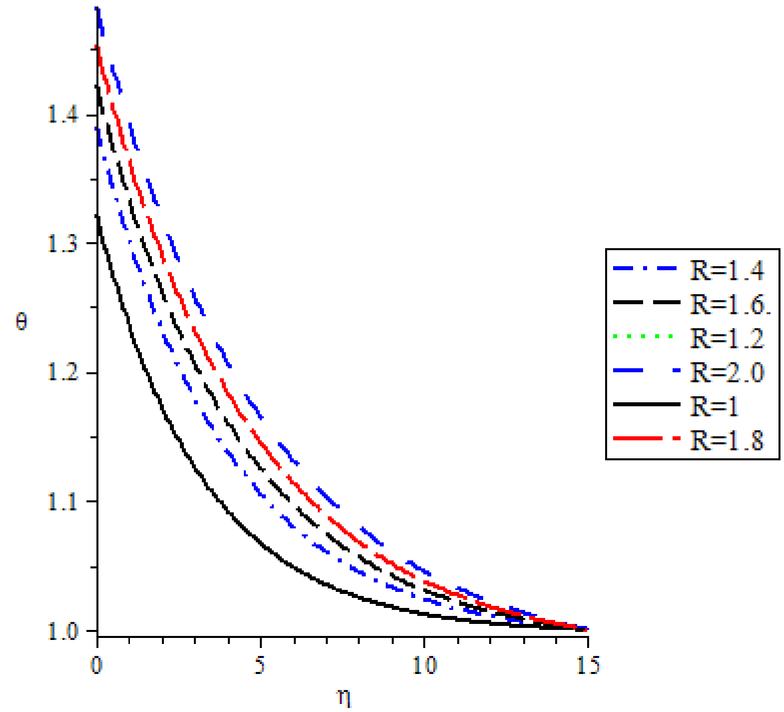

Fig.7: Temperature profile for various values of radiation number for
$Pr = 0.71$, $Gt = 1$, $Gc = 0.1$, $R = 1$, $\epsilon = 0.01$, $\alpha 2 = 0.1$, $Sc = 1$

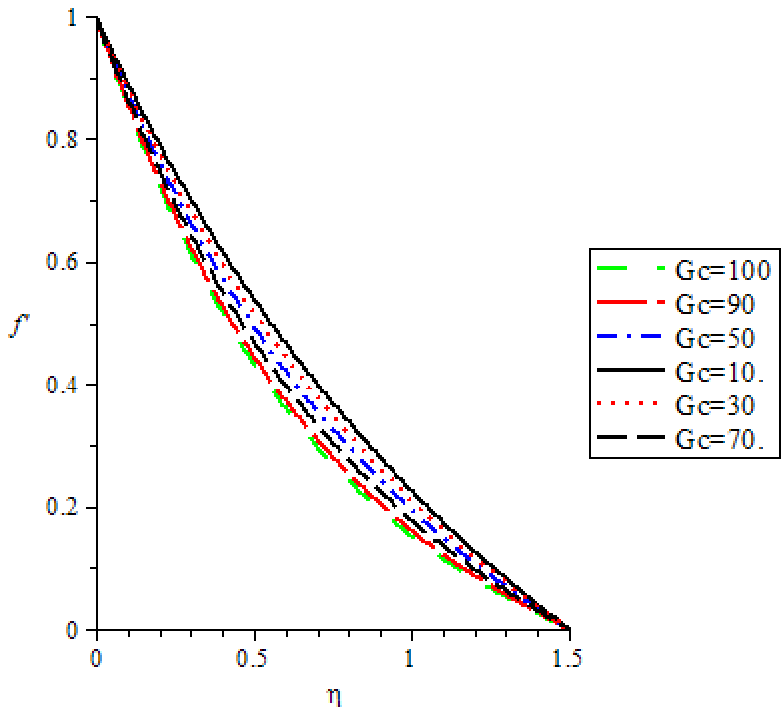

Fig.8: Velocity profile for various values of solutal Grassof number for
$Pr = 0.71$, $Gt = 1$, $Gc = 0.1$, $R = 1$, $\epsilon = 0.01$, $\alpha 2 = 0.1$, $Sc = 1$

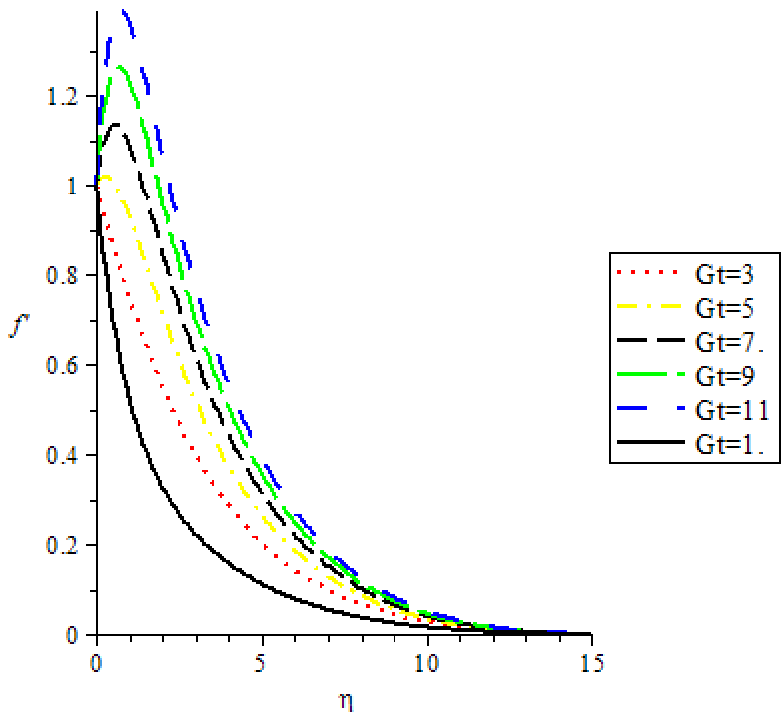

Fig.9: Velocity profile for various values of thermal Grassof number for
$Pr = 0.71$, $Gc = 0.1$, $R = 1$, $\epsilon = 0.01$, $\alpha 2 = 0.1$, $Sc = 1$

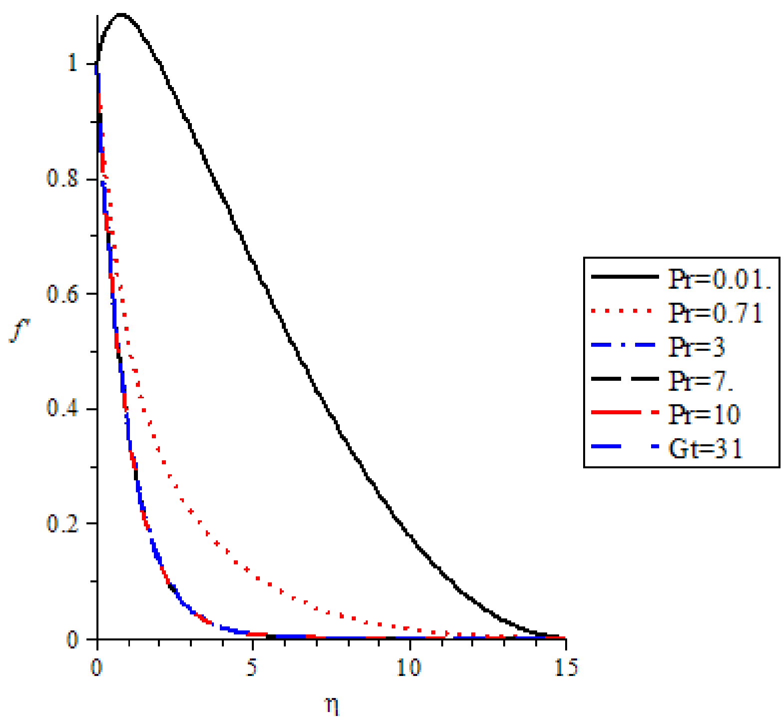

Fig.10: Velocity profile for various values of Prandtl number for
$Pr = 0.71$, $Gc = 0.1$, $R = 1$, $\epsilon = 0.01$, $\alpha 2 = 0.1$, $Sc = 1$

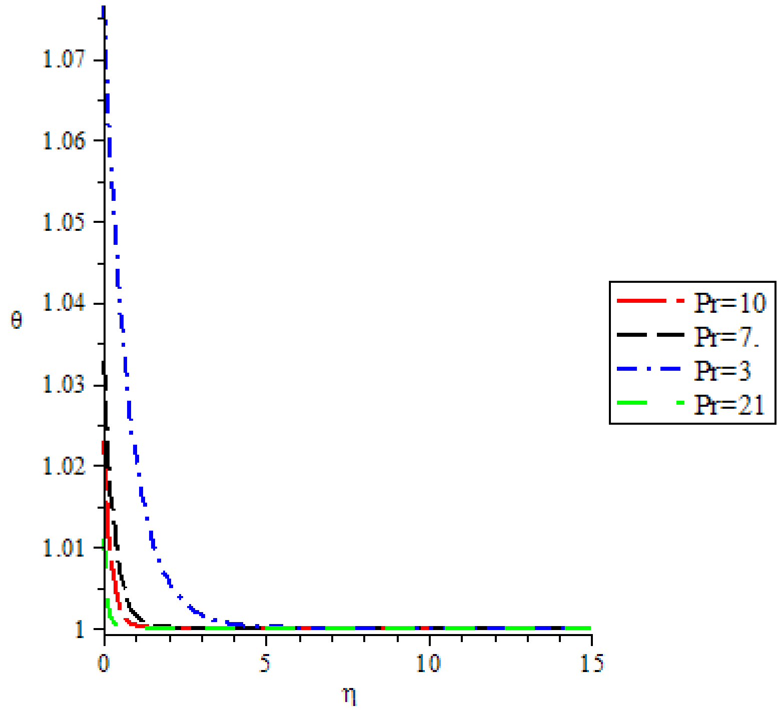

Fig 11: Temperature profile for various values of Prandtl number for
$Gt = 1, Gc = 0.1, R = 1, \epsilon = 0.01, \alpha 2 = 0.1, Sc = 1$

Conclusion

The problem of radiative double diffusive catalytic surface reaction has been solved using the method of similarity transformation combined with the fourth order Runge-Kuta shooting iterative technique. The effects of the various thermo-physical parameters have also been studied.

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