Scattering by Randomly Placed Perfectly Electromagnetic Conducting Random width Strip

Saeed Ahmed

Fazli Manan

Department of Electronics
Quaid-i-Azam University
45320 Islamabad
Pakistan.

Abstract

An analytic theory for the electromagnetic scattering from a perfect electromagnetic conducting (PEMC) randomly placed random width strip is developed, using the duality transformation which was introduced by Lindell and Sihvola. The theory allows for the occurrence of cross-polarized fields in the scattered wave, a feature which does not exist in standard scattering theory. This is why the medium is named as PEMC. PEMC medium can be transformed to perfect electric conductor (PEC) or perfect magnetic conductor (PMC) media. As an application, plane wave reflection from a planar interface of air and PEMC medium is studied. PEC and PMC are the limiting cases, where there is no cross-polarized component.

1 Introduction

The problem is concerned about random boundaries, i.e., scattering from half plane, strip or grating are very well known in the field of electromagnetics [1, 2, 3, 4]. Main aim is not to resolve these problems but just to introduce few random parameters in these planner boundaries for the PEC cases and to study the effects of the stochastic nature of these boundaries on the scattered field. Before to examine the random boundaries, i.e., scatterers with random parameters it is instructive to examine the behavior of randomly placed line source, because in two dimensional planner perfectly conducting boundaries, with sharp edges. An effort has been made to approximate edge diffraction by randomly placed pec random width strip, in far zone. In this paper, the solution for the following average scattered field by randomly placed random width strip has been transformed from pec case to pemc case.

2 Formulation of The Problem

Consider the case where random width strip is displaced randomly, along positive

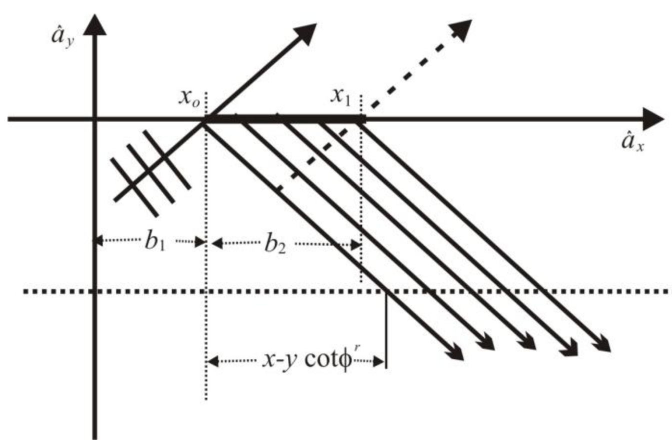

Figure 1: Geometry of the problem

x-axis, as in Fig.(1). In this case, location and width both are random. Let x_0 and x_1 are location of the edges on the positive x-axis, $x_0 = b_1$ and $x_1 = b_2$, where b_1 and b_2 are assumed to be statistically independent and identically distributed(i.i.d) random variables.

Here the distribution taken is an exponential distribution. The objective of the problem is to obtain the statistics of the scattered field from this random geometry in the far zone. The scattered field is again modelled as sum of reflected and diffracted fields, as given

$$E_z^s = E^r + E^d \quad (1)$$

In this case $f(x,y)$ will be modified as

$$f(x,y) = u(\tilde{x}-x)(1-u(\tilde{x}-x)) \quad (2)$$

Where $\tilde{x} = x - y\cot\phi^r$ and $u(x)$ are already defined. Therefore E^r can be written as

$$E^r = RE_0 e^{ikr\cos(\phi^r-\phi)} u(\tilde{x}-x_0)(1-u(\tilde{x}-x_1) \quad (3)$$

The diffracted field in this case modified as

$$E^d ; \; A(E_z^r(r=x_0)\frac{e^{ikr}}{\sqrt{kr}} + E_z^r(r=x_1)\frac{e^{ikr}}{\sqrt{kr}}) + O(k^{\frac{-3}{2}}) \quad (4)$$

Where $R_1 = \sqrt{(x-x_0)^2+y^2}$ and $R_2 = \sqrt{(x-x_1)^2+y^2}$. For large kr, far away from the strip R_1 and R_2 can be approximated as $R_1; \; r - x_0\cos\phi$ and $R_{21}; \; r - x_1\cos\phi$ respectively. The diffracted field can be written as

$$E^d ; \; A(e^{ikax_0} + e^{ikax_1})\frac{e^{ikr}}{\sqrt{kr}} + O(k^{\frac{-3}{2}}) \quad (5)$$

where $a = \cos\phi^i - \cos\phi$.

In order to find the statistics of scattered field, there is need to know the probability density function of the random variables x_0 and x_1. These random variables could be written as $x_0 = b_1$, $x_0 = b_1 + b_2$, where b_1 and b_2 are i.i.d random variables with exponential probability function. The marginal and joint probability functions of x_0 and x_1 are, [3]

$$P_{x_0}(x_0) = \lambda e^{-\lambda x_0}, x_0 \geq 0 \quad (6)$$

$$P_{x_1}(x_1) = \lambda^2 x_1 e^{-\lambda x_1}, x_1 \geq 0 \quad (7)$$

$$P_{x_0}x_1(x_0,x_1) = \lambda^2 e^{-\lambda x_1}, x_0 \geq 0, x_1 \geq x_0 \quad (8)$$

using these distributions, the statistics of the scattered field can be calculated the up to second order. The average scattered field is the sum of average reflected and diffracted fields and the variance of the scattered field could be written in terms of variance and covariances of reflected field and diffracted fields.

3 Average Reflected Field and its Variance

The reflected field is given, by taking the expectation value of the equation, the resulting expression is given below:

$$< E^r >= RE_0 e^{ikr\cos(\phi^r-\phi)} < u(\tilde{x}-x)(1-u(\tilde{x}-x)) > \quad (9)$$

Where the value of the expectation can be calculated as

$$< u(\tilde{x}-x_0)(1-u(\tilde{x}-x_1)) >=< u(\tilde{x}-x_0) > - < u(\tilde{x}-x_0)u(\tilde{x}-x_1) > \quad (10)$$

the average reflected field is

$$< E^r >= RE_0 e^{ikr\cos(\phi^r-\phi)} \lambda\tilde{x}e^{-\lambda\tilde{x}} u(\tilde{x}) \quad (11)$$

The second moment of reflected field will be

$$<| E^r |^2>= RE_0 < u(\tilde{x}-x)(1-u(\tilde{x}-x)) > \quad (12)$$

$$<| E^r |^2>= E_0^2 \lambda\tilde{x}e^{-\lambda\tilde{x}} \quad (13)$$

$$var(E^r) = E_0^2 \lambda \tilde{x} e^{-\lambda \tilde{x}} (1 - \lambda \tilde{x} e^{-\lambda \tilde{x}}) u(\tilde{x}) \quad (14)$$

The variance is maximum at $\lambda \tilde{x} = 1$. It can be seen that the strength of the field and its variance are equal for $\tilde{x} > 5/\lambda$.

4 Average Diffracted Field and its variance

The diffracted field due to random edge diffraction modelled by secondary sources, taking the expectation

$$< E^d >= A(< e^{ika\tilde{x}_0} > + < e^{ika\tilde{x}_1} >) \frac{ikr}{\sqrt{kr}} + O(k^{\frac{-3}{2}}) \quad (15)$$

The expression for average diffracted field is given as

$$< E^d >; \; A\left(\frac{1}{1 - ika < \tilde{x}_0 >} + (\frac{1}{1 - ika < \tilde{x}_0 >})^2 \right) \frac{e^{ikr}}{\sqrt{kr}} + O(k^{\frac{-3}{2}}) \quad (16)$$

Where x_0 and x_1, both are i.i.d, exponentially distributed. The variance of the diffracted field as

$$var(E^d) = (\frac{|A|^2}{kr})(\frac{(k^2a^2 < x_0 >^2)(5 + 2k^2a^2 < x_0 >^2)}{1 + k^2a^2 < x_0 >^2}^2) \quad (17)$$

5 Correlation and Covariance between Reflected and Diffracted fields

The cross correlation between E^r and E^d fields could be written as

$$\langle E^r E^{d^*} \rangle = RA^* E_0 e^{ikr\cos(\phi^r - \phi)} \frac{-ikr}{\sqrt{kr}} \langle (u(\tilde{x} - x_o) - u(\tilde{x} - x_1))(e^{-ikax_o} + e^{-ikax_1}) \rangle \quad (18)$$

$$< E^r E^{d^*} >= RA^* E_0 e^{ikr\cos(\phi^r - \phi)} (\frac{e^{-ikr}}{\sqrt{kr}})(\frac{-\lambda \tilde{x} e^{-(\lambda) - ika} \tilde{x}}{1 + ika < x_0 >} + \lambda e^{-(\lambda) - ika/2\tilde{x}} \frac{2}{ka} \sin(\frac{ka}{2}\tilde{x})) u(\tilde{x}) \quad (19)$$

We can write expression for covariance of reflected and diffracted fields as,

$$cov(E^r E^d) = RA * E_0 e^{ikr\cos(\phi^r - \phi)} \frac{e^{-ikr}}{\sqrt{kr}} \times \lambda \hat{x} e^{-\lambda \hat{x}} u(\hat{x}) \times e^{-i\frac{ka}{2}\tilde{x}} \frac{\sin(\frac{ka}{2}\tilde{x})}{\frac{ka}{2}\tilde{x}} - \frac{e^{-ika\tilde{x}}}{1 + ika < x_0 >} - \frac{2 + ika < x_0 >}{(1 + ika < x_0 >)^2} \quad (20)$$

It is observed that average field and its variance are independent of ϕ. The above average scattered field can be transformed from perfectly electric conducting case to perfectly electromagnetic conducting case by the following theory. The Concept of PEMC introduced by Lindell and Sihvola [3, 4] is a generalization of both PEC and PMC. An analytic theory for the electromagnetic scattering from a PEMC plane where a line source has been placed randomly, is developed. The PEMC medium characterized by a single scalar parameter M, which is the admittance of the surface interface, where $M = 0$ reduces the PMC case and the limit $M \rightarrow \pm\infty$ corresponds to the perfect electric conductor (PEC) case. The theory allows for the occurrence of cross-polarized fields in the scattered wave in the scattered wave, a feature which does not exist in standard scattering theory. This means that PEC and PMC are the limiting cases, for which there is no cross-polarized component. Because the PEMC medium does not allow electromagnetic energy to enter, an interface of such a medium behaves as an ideal boundary to the electromagnetic field. At the surface of a PEMC media, the boundary conditions between PEMC medium and air with unit normal vector n, are of the more general form. Because tangential components of the E and H fields are continuous at any interface of two media, one of the boundary conditions for the medium in the air side is $n \times (H + ME) = 0$, because a similar term vanishes in the PEMC-medium side. The other condition is based on the continuity of the normal component of the D and B fields which gives another boundary condition as $n.(D - MB) = 0$.

Here, PEC boundary may be defined by the conditions

$$n\times E=0,\qquad n.B=0 \tag{21}$$

While PMC boundary may be defined by the boundary conditions

$$n\times H=0,\qquad n.D=0 \tag{22}$$

where M denotes the admittance of the boundary which is characterizes the PEMC. For $M=0$, the PMC case is retrieved, while the limit $M\rightarrow\pm\infty$ corresponds to the PEC case. Possibilities for the realization of a PEMC boundary have also been studied [5].

It has been observed theoretically that a PEMC material acts as a perfect reflector of electromagnetic waves, but differs from the PEC andthe PMC in that the reflected wave has a cross-polarized component.

The duality transformations of perfectly electric condutor (PEC) to PEMC have been studied by many researchers [3, 4, 5, 6, 7, 8, 9]. Here we present an analytic scattering theory for a PEMC step, which is a generalization of the classical scattering theory.

Applying a duality transformation which is known to transform a set of fields and sources to another set and the medium to another one. In its most general form, the duality transformation can be defined as a linear relation between the electromagnetic fields. The effect of the duality transformation can be written by the following special choice of transformation parameters:

$$\begin{pmatrix} E_d \\ H_d \end{pmatrix}=\begin{pmatrix} M\eta_0 & \eta_0 \\ \frac{-1}{\eta_0} & M\eta_0 \end{pmatrix}\begin{pmatrix} E \\ H \end{pmatrix} \tag{23}$$

has the property of transforming PEMC to PEC, while

$$\begin{pmatrix} E \\ H \end{pmatrix}=\frac{1}{(M\eta_0)^2+1}\begin{pmatrix} M\eta_0 & -\eta_0 \\ \frac{1}{\eta_0} & M\eta_0 \end{pmatrix}\begin{pmatrix} E_d \\ H_d \end{pmatrix} \tag{24}$$

has the property of transforming PEC to PEMC [4].

Following the above relations [3], the transformed equations become as

$$E^r=-\frac{1}{M^2\eta_0^2+1}\left[(-1+M^2\eta_0^2)E^i+2M\eta_0u_z\times E^i\right] \tag{25}$$

$$E_{sd}=-(M\eta_0E_s+\eta_0H_s) \tag{26}$$

$$H_{sd}=-\frac{1}{\eta_0}E_s+M\eta_0H_s \tag{27}$$

$$E_s=\frac{1}{(M\eta_0)^2+1}\left[M\eta_0E_{sd}-\eta_0H_{sd}\right] \tag{28}$$

$$E_s=\frac{1}{(M\eta_0)^2+1}\left[((M\eta_0)^2-1)E_s-2M\eta_0^2H_s\right] \tag{29}$$

$$E_s=\frac{1}{(M\eta_0)^2+1}\left[((M\eta_0)^2-1)E_s-2M\eta_0E_s\right] \tag{30}$$

Where E_s, H_s are transformed pemc average fields and E_{sd}, H_{sd} are average scattered elecric and magnetic fields respectively.

This means that, for a linearly polarized incident field E^i, the reflected field from a such a boundary has a both co-polarized component, while $u_z\times E^i$ is a cross-polarized component, in the general case.

For the PMC and PEC special cases ($M = 0$ and $M = \pm\infty$ respectively), the cross-polarized component vanishes. For the special PEMC case $M = \frac{1}{\eta_0}$, such that

$$(E^r = -u_z \times E^i) \tag{31}$$

which means that the reflected field appears totally cross-polarized. It is obvious theoretically that a PEMC material acts as a perfect reflector of electromagnetic waves, but differs from the PEC ($E^r + E^i) = 0$ and $H^r = H^i$) and PMC ($E^r = E^i$ and $H^r + H^i = 0$) in that the reflected wave has a cross-polarized component.

6 Concluding remarks

In this work, a plane wave scattering by a randomly placed perfectly electromagnetic conducting random width strip has been randomly placed, has been studied. The theory provides explicit analytical formulas for the electric and magnetic field. An other formulla has been derived for the relative contributions to the scattered fields of the co-polarized and the crosspolarized fields depend on parameter M . The cross-polarized scattered fields vanish in the PEC and PMC cases, and are maximal for $M = \pm 1$. In the general case,the reflected wave has both a co-polarized and a cross-polarized component. The above transformed solution presents an analytical theory for the scattering by a randomly placed perfect electromagnetic random width strip. It is clear from the above discussion that for $M \to \infty$ and $M \to 0$ correspond to the PEC and PMC respectively. Moreover, for $M = \pm 1$ the medium reduces to PEMC.

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