Scattering by Perfectly Electromagnetic Conducting Random Width Strip

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Abstract

An analytic theory for the electromagnetic scattering from a perfectly electromagnetic conducting (PEMC) random width strip is developed, using the duality transformation which was introduced by Lindell and Sihvola. The theory allows for the occurrence of cross-polarized fields in the scattered field, a feature which does not exist in standard scattering theory. That is why the medium is named as PEMC (perfect electromagnetic conductor). PEMC medium can be transformed to Perfect Electric Conductor (PEC) or Perfect magnetic Conductor (PMC) media. As an application, a plane wave reflection from a planar interface of air and PEMC medium is studied. PEC and PMC are the limiting cases, where there is no cross-polarized component.

1 Introduction

The problems we are considering, i.e., scattering from half plane, strip or grating are very well known in the field of electromagnetics. Our aim is not to resolve these problems but introduce few random parameters in these planner boundaries [1, 2, 3, 4]. A complete solution exists for the perfectly electric conducting case in literature, based on the following equations and conditions, and to study the effects of the stochastic nature of these boundaries on the scattered field. Before examine the random boundaries, i.e., scatterers with random parameters it is instructive to examine the behavior of random with strip, because in two dimensional planner perfectly conducting boundaries, with sharp edges.

2 Formulation

Consider two dimensional case; two parameters related to the strip are: location of strip and its width. Assume that strip lies in the xz-plane and its location is deterministic. Let the origin lies at the starting edge of the strip and in this case the exact width of the strip is not known, so it is determined probabilistically only, i.e., here b is a random variable of some known probability density function. Take the exponential distribution of random variable b; whose probability density function is already defined. Since the scattered field is dependent on b and due to random nature of b, only one realization of scattered field for one value of b is obtained. To find the statistics of scattered field at least up to second order, i.e.,the average scattered field and its variance. Since the scattered field is sum of reflected field and edge diffractions and according to statistical theory, the average scattered field is average reflected field and average diffracted fields. The variance of scattered field depends on the variance of reflected and diffracted fields and their cross-covariance. The following averages will be calculated.

3 Formulation of The Problem

Consider a uniform plane wave incidents upon an electric conducting strip of width b and infinite length along z-direction, as shown in the Fig.(1).

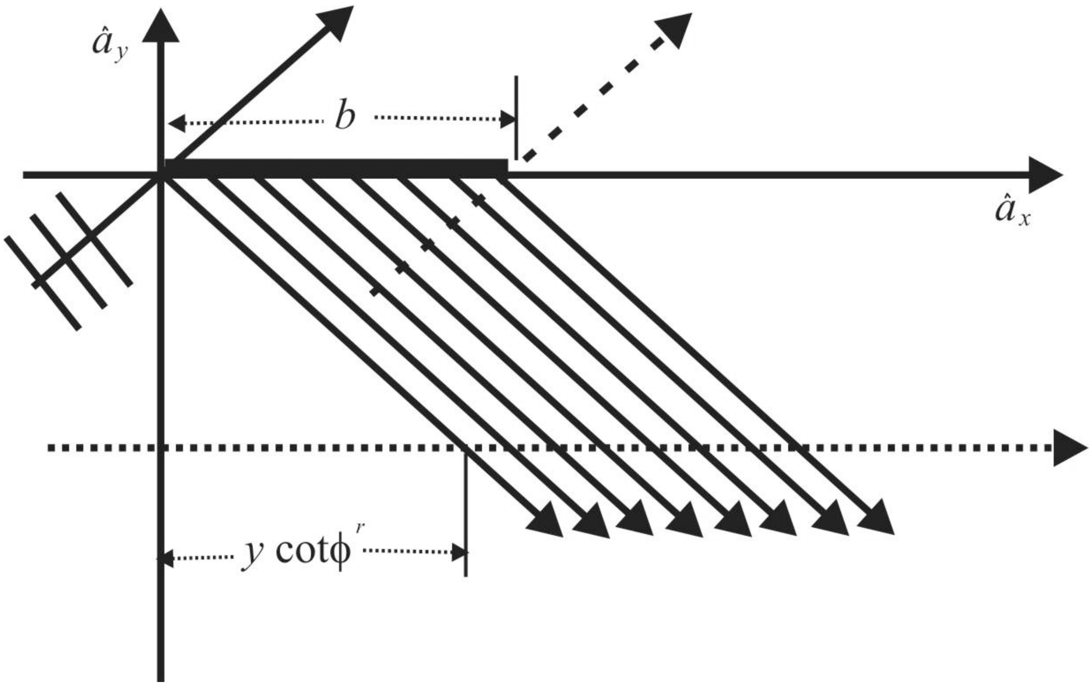

Figure 1: Geometry of the problem

The incident electric and magnetic field can be written as

$$E^i = \hat{a}_z E_o e^{ikr\cos(\phi^i - \phi)} \tag{1}$$

$$H^i = \frac{E_o}{\eta}(\hat{a}_x \sin\phi^i - \hat{a}_y \cos\phi^i) e^{ikr\cos(\phi^i - \phi)} \tag{2}$$

where E_o is a constant and it represents the amplitude of the incident electric field. The main interest is in the scattered field for the far zone only. The first thing is to model the scatter field, as sum of reflected field and edge diffractions. It can be described as,

$$E_z^s = E^r + E^d \tag{3}$$

where E^r is observed only in shaded region, as shown in Fig.(1). We are modelling the edge diffraction in the far zone, by field radiated by a line sources placed at edge locations. We modelled the current density induced on the surface of the strip by constant current plus two delta sources. The reflected field at xz-plane parallel to strip at y-depth could be represented by,

$$E^r = RE_o f(x,y) e^{ik(x\cos\phi^r + y\sin\phi^r} = RE_o f(x,y) e^{ik\cos(\phi^r - \phi)} \tag{4}$$

where $\phi^r = 2\pi - \phi^i$, as the reflected field is observed in the shaded region only, therefore $f(x,y)$ is 1 inside the shaded region but 0 outside this region.

$$f(x,y) = u(\tilde{x})1 - u(\tilde{x} - b) \tag{5}$$

where $u(x)$ is the unit step.

4 Average Reflected Field and Its variance

Taking the average of both sides of the reflected field:

$$< E^r >= RE_0 e^{ikr\cos(\phi^r - \phi)} < f(x,y) > \tag{6}$$

where $< f(x,y) >= u(\tilde{x})(1 - < u(\tilde{x} - b) >)$, using the exponential distribution of b, the average $< u(\tilde{x} - b) >$ can be calculated as,

$$< u(\tilde{x} - b) >= \int_{-\infty}^{\tilde{x}} P_b(b)db = 1 - e^{-\lambda\tilde{x}} \quad (7)$$

also we have

$$< f(x, y) >= u(\tilde{x})e^{-\lambda\tilde{x}} \quad (8)$$

Therefore average reflected field will become

$$< E^r >= RE_0 e^{-\lambda\tilde{x}} e^{ikr\cos(\phi^r - \phi} u(\tilde{x} \quad (9)$$

The variance of reflected field as,

$$var(E^r) = E_0^2 e^{-\lambda\tilde{x}}(1 - e^{-\lambda\tilde{x}})u(\tilde{x}) \quad (10)$$

From the above solution, it can be observed that, on the average reflected field is a plane wave, its strength decays exponentially along the positive x-direction in any plane parallel to the strip at depth y, in the shaded region. The variance of reflected field field first increases and is maximum at $\tilde{x} = ln2/\lambda$ and then it decreases to zero. The variation of average reflected field and its variance with respect to $\tilde{x}$ are shown in [?]. that the strength of field and its variance are equal for $\tilde{x} > 5\lambda$,

5 Average Diffracted Field and Its variance

The average diffracted field can be written by taking the statistical average of equation as,

$$< E^d >= A(1+< e^{ikab} >)\frac{e^{ikr}}{\sqrt{kr}} + O(k^{\frac{-3}{2}} \quad (11)$$

where the expected term $< e^{ikab} >$ can be calculated, by using exponential distribution, as

$$< e^{ikab} >= \int_{-\infty}^{\infty} e^{ikab} P_b(b)db = \lambda\int_0^{\infty} e^{(-\lambda + ika)b}db = \frac{1}{1 - ika<b>} \quad (12)$$

hence, the average diffracted field is

$$< E^d >= A(\frac{2 - ika<b>}{1 - ika<b>}\frac{e^{ikr}}{\sqrt{kr}} + O(k\frac{-3}{2}) \quad (13)$$

The variance of diffracted field is given below.

$$var(E^d) = |A|^2(\frac{k^2a^2<b>^2}{1 + k^2a^2<b>^2})\frac{1}{kr} \quad (14)$$

consider $\phi^i = \pi/4$, then for by increasing $< b >$, the average length of strip, a delta function comes out in the directions of incident and reflected wave. This is due to the fact that for large average width strip that can model the diffracted field as field radiated by perfectly conducting plate having slowly decaying exponentially distributed surface current density with travelling phase. The variance of diffracted field is zero in the directions of incident and reflected wave.

6 Correlation between Reflected and Diffracted fields

The correlation between the reflected and diffracted fields $< E^r E^d >$, is calculated as

$$< E^r E^d >= A^* RE_0 e^{ikr\cos(\phi^r - \phi)}\frac{e^{-ikr}}{\sqrt{kr}} u(\tilde{x}) < (1 + e^{-ikab})(1 - u(\tilde{x} - b) > \quad (15)$$

where $\tilde{x} = x - y\cot\phi^r$. The averaged term in the above expression can be calculated as,

$$< (1 + e^{-ikab})(1 - u(\tilde{x} - b)) >= 1+< e^{-ikab} > -< u(\tilde{x} - b) > -< e^{-ikab} u(\tilde{x} - b) > \quad (16)$$

where

$$< e^{-ikab} >= \frac{1}{1 + ika<b>} \quad (17)$$

$$< u(^x - b) >= 1 - e^{\lambda}\tilde{x} \quad (18)$$

$$< u(^{x} - b) > - < e^{-ikab}\, u(\tilde{x} - b) >= \frac{1 - e^{\lambda - ika}\,\tilde{x}}{1 + ika < b >} \tag{19}$$

using values of the averages in above equation, the expression for covariance of E^{r} and E^{d} can be written as

$$cov(E^{r}E^{d}) = A^{*}RE_{0}e^{ikr\cos(\phi^{r} - \phi)}\frac{e^{\lambda x}}{\sqrt{kr}}\frac{\left(e^{-ika\,\tilde{x}} - 1\right)}{1 + ika < b >}u(\mathrm{x}) \tag{20}$$

The average scattered field, sum of average diffracted and average reflected field, can be written by using the above equations

$$< E_{z}^{s} >= RE_{0}e^{ikr\cos(\phi^{r} - \phi)}e^{\lambda\tilde{x}}u(\tilde{\mathrm{x}}) + A\left(\frac{2 - ika < b >}{1 - ika < b >}\right)\frac{e^{ikr}}{\sqrt{kr}} + O\left(k\frac{-3}{2}\right) \tag{21}$$

The variance of scattered field in terms of variances and covariances of reflected and diffracted fields could be written as

$$var(E_{z}^{s}) = var(E^{r}) + var(E^{d}) + 2Rcov(E^{r}, E^{d}) \tag{22}$$

$$< E_{z}^{s} >= E_{0}^{2}e^{-\lambda\tilde{x}}u(\tilde{x}) + |A|^{2}\frac{k^{2}a^{2}<b>^{2}}{1 + (k^{2}a^{2}<b>^{2})}\frac{1}{kr} + 2R\left(A^{*}RE_{0}e^{ikr\cos(\phi^{r} - \phi}\frac{e^{-ika\,\tilde{x}}}{\sqrt{kr}}e^{\lambda(\tilde{x})}\left(\frac{\left(e^{-ika\,\tilde{x}} - 1\right)}{1 + ika < b >}\right)u(\tilde{x})\right) \tag{23}$$

The above average scattered field ($< E_{z}^{s} >=< E^{r} > + < E^{d} >$) can be transformed from perfectly electric conducting case to perfectly electromagnetic conducting case by the following theory. The Concept of PEMC introduced by Lindell and Sihvola [3, 4] is a generalization of both PEC and PMC. An analytic theory for the electromagnetic scattering by a perfectly electromagnetic conducting random width strip, is developed. The PEMC medium characterized by a single scalar parameter M, which is the admittance of the surface interface, where $M = 0$ reduces the PMC case and the limit $M \to \pm\infty$ corresponds to the perfect electric conductor (PEC) case. The theory allows for the occurrence of cross-polarized fields in the scattered wave in the scattered wave, a feature which does not exist in standard scattering theory. This means that PEC and PMC are the limiting cases, for which there is no cross-polarized component. Because the PEMC medium does not allow electromagnetic energy to enter, an interface of such a medium behaves as an ideal boundary to the electromagnetic field. At the surface of a PEMC media, the boundary conditions between PEMC medium and air with unit normal vector n, are of the more general form. Because tangential components of the E and H fields are continuous at any interface of two media, one of the boundary conditions for the medium in the air side is $n \times (H + ME) = 0$, because a similar term vanishes in the PEMC-medium side. The other condition is based on the continuity of the normal component of the D and B fields which gives another boundary condition as $n.(D - MB) = 0$.

Here, PEC boundary may be defined by the conditions

$$n \times E = 0, \qquad n.B = 0 \tag{24}$$

While PMC boundary may be defined by the boundary conditions

$$n \times H = 0, \qquad n.D = 0 \tag{25}$$

where M denotes the admittance of the boundary which is characterizes the PEMC. For $M = 0$, the PMC case is retrieved, while the limit $M \to \pm\infty$ corresponds to the PEC case. Possibilities for the realization of a PEMC boundary have also been studied [5].

It has been observed theoretically that a PEMC material acts as a perfect reflector of electromagnetic waves, but differs from the PEC andthe PMC in that the reflected wave has a cross-polarized component.

The duality transformations of perfectly electric condutor (PEC) to PEMC have been studied by many researchers [3, 4, 5, 6, 7, 8, 9]. Here we present an analytic scattering theory for a PEMC step, which is a generalization of the classical scattering theory.

Applying a duality transformation which is known to transform a set of fields and sources to another set and the medium to another one. In its most general form, the duality transformation can be defined as a linear relation between the electromagnetic fields. The effect of the duality transformation can be written by the following special choice of transformation parameters:

$$\begin{pmatrix} E_d \\ H_d \end{pmatrix} = \begin{pmatrix} M\eta_0 & \eta_0 \\ \frac{-1}{\eta_0} & M\eta_0 \end{pmatrix} \begin{pmatrix} E \\ H \end{pmatrix} \quad (26)$$

has the property of transforming PEMC to PEC, while

$$\begin{pmatrix} E \\ H \end{pmatrix} = \frac{1}{(M\eta_0)^2+1} \begin{pmatrix} M\eta_0 & -\eta_0 \\ \frac{1}{\eta_0} & M\eta_0 \end{pmatrix} \begin{pmatrix} E_d \\ H_d \end{pmatrix} \quad (27)$$

has the property of transforming PEC to PEMC [4].

Following the above relations [3], the transformed equations become as

$$E^r = -\frac{1}{M^2\eta_0^2+1}\left[(-1+M^2\eta_0^2)E^i + 2M\eta_0 u_z \times E^i\right] \quad (28)$$

$$E_{sd} = -(M\eta_0 E_s + \eta_0 H_s) \quad (29)$$

$$H_{sd} = -\frac{1}{\eta_0}E_s + M\eta_0 H_s \quad (30)$$

$$E_s = \frac{1}{(M\eta_0)^2+1}\left[M\eta_0 E_{sd} - \eta_0 H_{sd}\right] \quad (31)$$

$$E_s = \frac{1}{(M\eta_0)^2+1}\left[((M\eta_0)^2-1)E_s - 2M\eta_0^2 H_s\right] \quad (32)$$

$$E_s = \frac{1}{(M\eta_0)^2+1}\left[((M\eta_0)^2-1)E_s - 2M\eta_0 E_s\right] \quad (33)$$

Where E_s, H_s are transformed pemc average fields and E_{sd}, H_{sd} are average scattered elecric and magnetic fields respectively.

This means that, for a linearly polarized incident field E^i, the reflected field from a such a boundary has a both co-polarized component, while $u_z \times E^i$ is a cross-polarized component, in the general case. For the PMC and PEC special cases ($M = 0$ and $M = \pm\infty$ respectively), the cross-polarized component vanishes. For the special PEMC case $M = \frac{1}{\eta_0}$, such that

$$(E^r = -u_z \times E^i) \quad (34)$$

which means that the reflected field appears totally cross-polarized. It is obvious theoretically that a PEMC material acts as a perfect reflector of electromagnetic waves, but differs from the PEC $(E^r + E^i) = 0$ and $H^r = H^i$) and PMC ($E^r = E^i$ and $H^r + H^i = 0$) in that the reflected wave has a cross-polarized component.

7 Concluding remarks

In this work, a plane wave scattering by perfectly electromagnetic conducting random width strip is studied. The theory provides explicit analytical formulas for the electric and magnetic field. An other formulla has been derived for the relative contributions to the scattered fields of the co-polarized and the crosspolarized fields depend on parameter M. The cross-polarized scattered fields vanish in the PEC and PMC cases, and are maximal for $M = \pm 1$. In the general case,the reflected wave has both a co-polarized and a cross-polarized component. The above transformed solution presents an analytical theory for the scattering by perfectly electromagnetic conducting random width strip. It is clear from the above discussion that for $M \to \infty$ and $M \to 0$ correspond to the PEC and PMC respectively. Moreover, for $M = \pm 1$ the medium reduces to PEMC.

8 Acknowledgements

The author would like to acknowledge the Quaid-i-azam University Islamabad Pakistan for providing research environment and moral support.

References

[1] Atif Raza. Scattering from Random Boundaries. M.Phil. Thesis, Department of Electronics, Quaid-i-Azam Univerisity, Islamabad, 2003.

[2] B. Senadji and A. J. Levy, A Statistical Model for the Simulation of Time Varying Multipath Mobile Radio Propagation Channel, IEEE International Conference on Acoustics, Speech and Signal Processing, ICASSP-94, 1994.

[3] Athanasios Papoulis, Probability, Random Variables, and Stochastic Processes, McGraw-Hill, New York, 1991.

[4] Helstrom, C. W., Probability and Stochastic Processes for Engineers, McMillan, 1990.

[5] Lindell, I. V. and A. H. Sihvola, Perfect electromagnetic conductor, Journal of Electromagnetic Waves and Applications, Vol. 19, 861-869, 2005.

[6] Lindell, I. V. and A. H. Sihvola, Transformation method for problems involving perfect electromagnetic conductor (PEMC) structures, IEEE Trans. on Ant. and Propag., Vol. 53, 3005-3011, 2005.

[7] Lindell, I. V. and A. H. Sihvola, Realization of the PEMC boundary, IEEE Trans. on Ant. and Propag., Vol. 53, 3012-3018, 2005.

[8] Lindell, I. V., Electromagnetic fields in self-dual media in differential-form repre-sentation, Progress In Electromagnetics Research, PIER 58, 319-333, 2006.

[9] Ruppin, R., Scattering of electromagnetic radiation by a perfectelectromagnetic conductor cylinder, Journal of Electromagnetic Waves and Applications, Vol. 20, No. 13, 1853-1860, 2006.

[10] Lindell, I. V. and A. H. Sihvola, Reflection and transmission of waves at the interface of perfect electromagnetic conductor (PEMC), Progress In Electromagnetics Research B, Vol. 5, 169-183, 2008.

[11] M. A. Fiaz, B. Masood, and Q. A. Naqvi, "Reflection From Perfect Electromagnetic Conductor (PEMC) Boundary Placed In Chiral Medium", J. of Electromagn. Waves and Appl., Vol. 22, 1607-1614, 2008.