Comparison of Numerical Techniques for Coordinate Transformation: The Case Study of Nigeria Transverse Mercator and Universal Transverse Mercator

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Abstract

In this paper, numerical techniques were employed to transform coordinates from the Nigerian Transverse Mercator (NTM) system to Universal Transverse Mercator (UTM) system. The two numerical techniques used are Newton's divided difference and cubic spline techniques. The data used are the simulated NTM coordinates of points in the West belt of NTM projection system which also fall in zone 31 of UTM projection system. All the computations were carried out on Pentium IV computer using computer program written in Visual Basic language. These computations were based on rectangular coordinate system. This is because both NTM and UTM are based on plane surveying system. Based on the results obtained and comparison of the two numerical techniques with the standard step-wise analytical technique, it can be inferred that the numerical techniques serve as effective tools for coordinate transformation from NTM system to UTM system. However, cubic spline technique proved to be more accurate and reliable but less economical of computing time and computer memory spaces than the Newton's divided difference technique.

Keywords: Coordinate transformation, Newton's divided difference technique, Cubic spline technique, Step-wise analytical technique, NTM and UTM

Introduction

The United State of America Army Corps of Engineers in 1940s suggested the need for a world wide plane coordinate system which later resulted in the development and adoption of the Universal Transverse Macerator (UTM) system (USGS, 2001). The aim of this was to have a common international reference system with universal acceptability and to unify world wide grid system so as to facilitate global scientific operations and projects execution. Consequently, the Federal Government of Nigeria introduced UTM system, around 1975, for surveying and mapping. This was to replace the Nigerian Transverse Macerator (NTM) system, which had been in use. This, therefore, necessitated the transformation of coordinates of points in NTM system to the equivalent coordinates of those points in UTM system. Previous attempts made by Edoga (1979), Stuifbergen (2009) and Karney (2011), using analytical techniques, to accomplish the task of coordinate transformation has been successful. However, investigations revealed that quite a number of states in Nigeria are lukewarm in the enforcement of the change over to UTM system. While some states still use NTM system, others maintain both NTM and UTM systems.

This might not be unconnected with the fact that analytical techniques are laborious, uneconomical and difficult to understand in approach even though they produce accurate results in a continuous domain. Realising the need to search for an alternative technique, Idowu (1996) succeeded in using a numerical technique to solve the coordinate transformation problems. The simplicity in approach and the satisfactory results obtained using this technique have stimulated investigations into other numerical techniques for coordinate transformation. Therefore, it is the objective of this paper to transform coordinates of points from NTM system to UTM system using Newton's divided difference numerical technique and Cubic spline technique. Comparisons of the two numerical (i.e. Cubic Spline and Newton's divided difference) techniques are made with the standard step-wise analytical technique to determine the adequacy of each of the numerical techniques in order to know which of them is better than the other under given circumstances.

Transverse Mercator, Nigerian Transverse Mercator and Universal Transverse Mercator

For mapping purposes, Surveyors need to graphically represent a portion of the earth by a plane map sheet of the area. Map projection is the process of producing this graphical representation of curved surface (e.g. portion of the earth) on a plane surface (e.g. paper) in form of maps/plans and to express the portion of discrete points on the curved surface on a plane surface in order to simplify the computation of distances and directions within the system of such discrete points.

There are limitless numbers of different map projections (e.g. azimuthal, obligue, cylindrical, conformal projections etc.) but a universally accepted projection system (except for polar regions) is the Transverse Mercator (TM) projection. This can be described as conformal cylindrical transverse projection. Its design and properties are fully enumerated in Field (1980).

NTM is a modified version of TM adopted for Nigeria. The modifications take care of the large expanse of the country which covers about $l0^0$ (i.e. 4^0N - 14^0N) latitude and 12^0 (i.e. 2^0 30^0 - 14^0 30^0E) longitude. It is generally referred to as 3-belt system (Omoigui, 1973). UTM, developed for a world wide application apart from the polar zones, is also based on TM with more modifications to the TM. Its application is limited to between latitudes 80^0N and 80^0S. However, both polar zones are covered by the Universal Polar Stereographic (UPS) system which complements UTM but is quite independent of it (USGS, 2001) and Stuifbergen, 2009). The major characteristics of both NTM and UTM and the various modifications to TM and NTM are fully discussed in Fajemirokun and Nwillo (1990).

Transformation of Coordinates from NTM to UTM

NTM has been in use in Nigeria while UTM was introduced around 1975. Therefore, coordinate transformation from NTM system to UTM system has thus become a current and recurring problem for most surveyors. Coordinate transformation can be defined as the process of establishing the relationship between coordinate systems in order to transform points from one system to the other. Examples of these are transformation of coordinates from local origin to national origin, photo coordinates to ground coordinates and many others. There are various methods of coordinate transformation. These include projective transformation, affine transformation, conformal transformation etc. The choice of method, however, depends on the problem at hand. In this paper, conformal transformation, which has the property of shape preservation during transformation, is used to transform coordinates from NTM system to UTM system. This was considered appropriate because NTM and UTM are both products of conformal projection. Therefore, UTM coordinates (X, Y) can be expressed as mathematical functions of NTM coordinates (x, y). That is:

$$X = f_1(x,y)$$
$$Y = f_2(x,y) \qquad (1)$$

The functions (f_1 and f_2) are unique, reciprocal, finite and continuous (Idowu, 1996). Two major techniques of coordinate transformation exist. These are Analytical techniques and numerical techniques.

Analytical technique of coordinate transformation

Analytical technique, also called empirical or classical technique, is a procedure that permits the exact solution of mathematical problems in an infinite number of steps (i.e. a continuos domain). In the transformation of coordinates from NTM system to UTM system, two of these techniques available are step-wise technique and direct technique as enumerated by Edoga (1979) and Karney (2011) respectively.

Numerical technique of coordinate transformation

Numerical technique is a procedure that approximates a continuous function by a class of discrete functions. The advantage of this is the replacement of the complicated functions by some simpler functions so that many mathematical operations such as integration and differentiation can be more easily performed. However, its accuracy is relatively low due to the replacement of the continuous functions by discrete functions and non-exact solution of the discrete functions (i.e. approximating the number of decimal digits involved in the results). For satisfactory results, there must be numerical stability. That is, errors due to the reasons for relative low accuracy must be reduced to the barest minimum. There are various classes of numerical techniques such as rational function, trigonometric function, interpolating polynomial and so on.

Of these, interpolating polynomial is by far the most reliable and hence widely used (Conte and de Boor, 1972). In this study, two numerical techniques of interpolating polynomial are employed to transform coordinates from NTM system to UTM system. These techniques are Newton's divided difference technique and Cubic Spline technique.

Newton's divided difference technique

This is the Newton's numerical process of discretising a continuous function. Its mathematical functions given by Sen and Kirshnamurthy (1986) are discussed below:

$$P_n(X) = \Sigma_{i=0}^{n} \; f(x_0, x_1, x_{2\ldots}x_n) \; \Pi_{j=0}^{i} (x-x_j) \qquad (2)$$

where: $i, j = 0, 1, 2, 3, \ldots\ldots\ldots\ldots n$

i.e. $P_n(X) = f(x_0) + f(x_0,x_1)(x-x_0) + f(x_0,x_1,x_2)(x-x_0)(x-x_i) + \ldots + f(x_0,x_1,x_{2\ldots}x_n)(x-x_0)(x-x_1)(x-x_2)\ldots\ldots.(x-x_n)$.

$$f(x_0,x_1) = \frac{f(x_1) - f(x_0)}{x_1 - x_0} \qquad (3)$$

$$f(x_0,x_1,x_2) = \frac{f(x_1,x_2) - f(xo\text{-}x_1)}{x_2 - x_0} \qquad (4)$$

$$f(x_0,x_1,x_{2\ldots}x_n) = \frac{f(x_1,x_2\ldots x_n) - f(x_0,x_1,x_2\ldots x_{n-1})}{x_n - x_0} \qquad (5)$$

where: n = number of tabular points (i.e. number of stations whose coordinates are used for constructing the polynomial).

P(X) = Numerical values of UTM coordinates at x-NTM coordinates.

f(x) = Analytical values of UTM coordinates at x-NTM coordinates.

Using equation (3) to (5), all the divided difference values needed for the equation (2) can be generated as shown in Table 1.

This technique has an advantage of being applicable at equal or unequal interval (h) of arguments of functions.

where: $h_i = x_{i+1} - x_i$ (6)

Scarborough (1964) described the method as the Newton's general technique because at equal interval of argument (i. e. h = constant), it produces the popularly known Newton forward and backward difference interpolating polynomials. With unequal interval (i. e. h not equal to constant), it becomes lagranges interpolating polynomials. However, the degree of interpolating polynomial (r) for a particular problem at hand depends on the number of data steps (n) used in equation (2). This is given by:

$$r = n-1 \qquad (7)$$

Table 1: Divided Difference Table

X_i	$f(x_i)$	$f(x_i,x_{i+1})$	$f(x_i,x_{i+1}, x_{i+2})$	$f(x_i,x_{i+1}, x_{i+2,}\ldots x_n)$
X_0	$f(x_0)$			
		$f(x_0,x_1)$		
X_1	$f(x_1)$		$f(x_0,x_{1,}\ x_2)$	
		$f(x_1,x_2)$		$f(x_0,x_{1,}\ x_{2,}\ldots x_n)$
		"		
X_2	$f(x_2)$	"	$f(x_1,x_{2,\ldots}\ x_n)$	
"	"	"		
"	"	"		
"	"	$f(x_{n-1},x_n)$		
"	"			
"	"			
X_n	$f(x_n)$			

Cubic Spline Technique

Cubic spline technique, one of the so called better behaved numerical techniques, is a piecewise cubic polynomial with continuity up to and including its second derivatives. Although details of its mathematical procedures are fully discussed in Idowu (1996), the following major equations are repeated here for easy reference:

$$y = \alpha_i (x-x_i)^3 + \beta_i(x-x_i)^2 + \gamma_i(x-x_i) + \delta_i \quad (8)$$

$$y_i = \delta_i \quad (9)$$

$$\alpha_i = \frac{M_{i+1} - M_i}{6h_{i+1}} \quad (10)$$

$$\beta_i = M_i /2 \quad (11)$$

$$\gamma_i \; \frac{(y_{i+1} - y_i)}{h_{i+1}} - \frac{(2h_{i+1}M_i + h_{i+1}M_{i+1})}{6} \quad (12)$$

M_i is obtained by solving the system of linear equations given in matrix form below:

$$AM = D \quad (13)$$

Where:

$$A_{n+1,n+1} = \begin{matrix} h_2 & -(h_1+h_2) & h_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ h_1 & 2(h_1+h_2) & h_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & h_2 & 2(h_1+h_2) & h_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_3 & 2(h_3+h_4) & h_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & h_{n-1} & 0 & 0 & 0 & 0 & 2(h_{n-1}+h_n) & h_n \\ 0 & 0 & 0 & 0 & h_n & 0 & 0 & 0 & 0 & -(h_{n-1}+h_n) & h_{n-1} \end{matrix}$$

$$M_{n+1,1} = \begin{matrix} M_0 \\ M_1 \\ M_2 \\ `` \\ `` \\ `` \\ M_{n-1} \\ M_n \end{matrix} \qquad D_{n+1,1} = \begin{matrix} 0 \\ \frac{(y_2-y_1)}{h_2} - \frac{(y_1-y_0)}{h_1} \\ \frac{(y_3-y_2)}{h_3} - \frac{(y_2-y_1)}{h_2} \\ `` \\ `` \\ `` \\ \frac{(y_n-y_{n-1})}{h_n} - \frac{(y_{n-1}-y_{n-2})}{h_{n-1}} \\ 0 \end{matrix}$$

x_i = NTM coordinates of point i used for designing the polynomial
y_i = UTM coordinates of point i for x_i
x = NTM coordinates of point whose UTM coordinate is needed
y = UTM coordinates of point for x

Presentation of Data

The polynomials designed for the two numerical techniques were constructed using data shown in Table 2. Columns 1 and 2 of the table show the simulated NTM coordinates chosen in the west belt of NTM system and zone 31 of UTM system while columns 3 and 4 show the equivalent UTM values of these coordinates. These UTM values were obtained using the analytical technique of Edoga (1979).

Table 2: Data for Designing Polynomials

NTM COORDINATES		UTM COORDINATES	
Easting (m)	**Northing (m)**	**Easting (m)**	**Northing (m)**
215000.000	346000.000	649915.105	788252.837
221000.000	352000.000	655894.821	794273.201
227000.000	358000.000	661874.535	800293.361
233000.000	364000.000	667854.247	806314.815
239000.000	370000.000	674833.957	812336.065
245000.000	376000.000	679813.664	818357.610
251000.000	382000.000	685793.370	824379.450
257000.000	388000.000	691773.073	830401.585
263000.000	394000.000	697752.774	836424.015

Presentation of Results

Divided difference technique yields the results shown in Tables 3 and 4 while cubic spline results are presented in Tables *5* and 6. Columns 1 of these Tables show the NTM coordinates used to test the suitability of the numerical techniques, columns 2 give the UTM (analytical) coordinates equivalent of columns 1, columns 3 show the UTM (numerical) coordinates equivalent of columns 1 while the difference between the results obtained by the analytical technique and the numerical techniques are presented in columns 4. Other parameters used for further comparison of the techniques are shown in Table 7.

Table 3: Divided Difference Easting Coordinates

	EASTING COORDINATES (m)		
NTM	**UTM (Analytical)**	**UTM (Numerical)**	**UTM(Aly Num.)**
218000.000	652904.964	652904.963	0.001
221000.000	655894.821	655894.821	0.000
224000.000	658884.678	658884.678	0.000
227000.000	661874.535	661874.535	0.000
230000.000	664864.391	664864.392	0.001
233000.000	667854.247	667854.247	0.000
236000.000	670844.102	670844.102	0.000
239000.000	673833.957	673833.957	0.000
242000.000	676823.811	676823.811	0.000

Table 4: Divided Difference Northing Coordinates

	NORTHING COORDINATES (m)		
NTM	**UTM (Analytical)**	**UTM (Numerical)**	**UTM(A1y. - Num.)**
249000.000	791262.982	791262.982	0.000
352000.000	794273.201	794273.201	0.000
355000.000	797283.494	197283.494	0.000
358000.000	800293.861	800293.861	0.000
361000.000	803304.301	803304.302	-0.001
364000.000	806314.815	806314.817	-0.002
367000.000	809325.403	809325.406	-0.003
370000.000	812336.065	812336.069	-0.004
373000.000	815346.801	815346.805	-0.004

Table 5: Cubic Spline Easting Coordinates

NTM	EASTING COORDINATES (m) UTM (Analytical)	UTM (Numerical)	UTM (AIy. - Num.)
218000.000	652904.964	652904.963	0.001
221000.000	655894.821	655894.821	0.000
224000.000	658884.678	658884.678	0.000
227000.000	661874.535	661874.535	0.000
230000.000	664864.391	664864.391	0.000
233000.000	667854.247	667854.247	0.000
236000.000	670844.102	670844.102	0.000
239000.000	673833.937	673833.957	0.000
242000.000	676823.811	676823.811	0.000

Table 6: **Cubic Spline Northing Coordinates**

NTM	NORTHING COORDINATES(m) UTM (Analytical)	UTM (Numerical)	UTM(Aly. Num.)
349000.000	791262.982	791262.982	0.000
352000.000	794273.201	794273.201	0.000
355000.000	797283.494	797283.494	0.000
358000.000	800293.861	800293.861	0.000
361000.000	803304.301	803304.301	0.000
364000.000	806314.815	806314.815	0.000
367000.000	809325.403	809325.403	0.000
370000.000	812336.065	812336.065	0.000
373000.000	815346.801	815346.801	0.000

Table 7: Other Parameters used for General Comparison

Other Parameters	Analytical Technique	Numerical Technique Cubic Spline	Divided Difference
Computer storage (Bytes)	6294.00	9308.00	3689.00
Computer Tinie (seconds)	46.03	38.18	12.14
Degree of Polynomial	N. A.	3 (constant)	varies

Analysis and comparison of results

The results show that the UTM coordinates obtained by the analytical technique and numerical techniques compare favourably well. This indicates the high level of suitability of the numerical techniques for coordinate transformation. However, taking analytical technique as standard, it is observed from Tables 4 and 6 that cubic spline technique produced relatively smaller coordinate differences than the divided difference technique. This tends to confirm cubic spline technique as a better behaved, more accurate and reliable than other numerical technique.

Further investigations show that if more number of data step points are used, the accuracy of cubic spline technique reduces randomly but insignificantly while that of divided difference technique reduces progressively and significantly. This seems to follow a progressive error pattern increase of 0.00lm as the coordinates to be transformed increases by 3000.000mm. This is because increase in the number of data step points does not increase the cubic spline degree of polynomial hence the effect of round off error increase but negligibly. With the divided difference technique, however, the more the number of data step points used, the higher the degree of polynomial which allows the accumulation of round off errors and hence poorer results.

If the step length is small, the two numerical techniques generally yield acceptable results. Also, It is noted, in Table 7, that computer time and memory spaces used for the cubic spline technique is more than that of divided difference technique. Based on the fact that the cost of project executed by computer is directly proportional to its time and memory spaces, it can be inferred that divided difference technique is more economical than the cubic spline technique.

Conclusions

In this paper, numerical techniques have been used to transform coordinates from NTM system to UTM system. The two numerical techniques used are the Newton's divided difference technique and Cubic spline technique. The insignificant disagreement between the analytical results and the numerical results shows that the numerical techniques satisfactorily transform coordinates from NTM system to UTM system. The two numerical techniques can effectively transform coordinates from NTM system to UTM system and vice versa within the data range used for designing the polynomials. With more than the number of data points used for the construction of polynomial, the accuracy of the cubic spline technique reduces randomly but insignificantly while that of divided difference decreases progressively and significantly. Therefore, cubic spline technique proves to be more accurate and reliable than the divided difference technique. However, the divided difference technique is simpler, more cost-effective than other numerical technique.

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