General Fault Admittance Method Line-to-Ground Faults in Reference and Odd Phases

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Abstract

Line-to-ground faults are usually analysed using symmetrical components. As a first step, a reference phase is chosen which results in the simplest connection of the symmetrical component sequence networks for the fault. The simplest connection of symmetrical component sequence networks is a series one when the line-to-ground fault is in the reference phase, say phase a of an a b c phase system. Putting the fault on an odd phase results in series connections of sequence networks that involve phase shifts, and the solution is more demanding. In practice, the results for the fault in the reference phase may be translated to the odd phase by appropriate substitution of phases. In this approach, the solution proceeds by assuming that the fault is in the reference phase and that the symmetrical sequence networks are connected in series. The series connection of the sequence networks at the fault point is solved for the symmetrical component currents and voltages. These are then used to determine the symmetrical component voltages at the other busbars and hence the symmetrical component currents in the rest of the system. The connection of the sequence networks must be known for the common fault types. In contrast, a solution by the general method of fault admittance matrix does not require prior knowledge of how the sequence networks are connected. A line-to-ground fault may be on any phase, reference or odd, and a solution is obtained for the particular fault. It is therefore more versatile than the classical methods in that it does not depend on prior knowledge of how the sequence networks are connected. The paper presents solutions for line-to-ground faults on the reference and odd phases of a simple power system containing a delta-earthed star connected transformer. The results, which include the effects of the delta-earthed star connected transformer, show that the general fault admittance method can be used to solve line-to-ground faults on odd phases.

Keywords: Line-to-ground fault on odd phases, Unbalanced faults analysis, Fault admittance matrix, Delta-earthed-star transformer.

1. Introduction

The paper shows that the general fault admittance method of fault analysis allows solution of single to ground faults on odd phases. The method does not require one to have a good understanding of how the sequence networks are connected as in the classical approach, so that one may interpret the results obtained for the reference phase fault to the odd phases.

The general fault admittance method differs from the classical approaches based on symmetrical components in that it does not require prior knowledge of how the sequence components of currents and voltages are related. In the classical approach, knowledge of how the sequence components are related is required because the sequence networks have to be connected in a prescribed way for a particular fault. Then the sequence currents and voltages at the fault are determined, after which symmetrical component currents and voltages in the rest of the network are calculated. Phase currents and voltages are found by transforming the respective symmetrical component values [2-9].

Another consideration is that in the classical analysis, common faults have reference faults that are solved and then the results applied to odd phase faults. For example line-to-ground faults are always solved with reference to the *a* phase, in an *abc* phase system, or its equivalent.

It is known that for this type of fault $V_1 + V_2 + V_0 = 0$ and that $I_1 = I_2 = I_0$, where the variables V and I refer to voltage, current respectively, and the subscripts *1*, *2* and *0* refer to the positive, negative and zero sequence components respectively.

The solution is therefore obtained with reference to the reference phase. However, when the line-to-ground fault is on the odd phase, either the *b* or *c* phase, the results for the reference phase fault must be interpreted in respect of the odd phase, taking into account the symmetrical component constraints. The reference phase is replaced by the odd phase and the results converted accordingly. Table 1 shows the voltage and current symmetrical component constraints for a line-to-ground faults on the reference *a* and odd phases *b* and *c*.

Table 1: Symmetrical Component Constraints for Line-to-Ground Fault.

Reference Phase	Odd Phases	
a	*b*	*c*
$V_{a1}+V_{a2}+V_{ao}=$ $V_1+V_2+V_0=0$	$V_{b1}+V_{b2}+V_{bo}=$ $\alpha^2V_{a1}+\alpha V_{a2}+V_{a0}=$ $\alpha^2V_1+\alpha V_2+V_0=0$	$V_{c1}+V_{c2}+V_{co}=$ $\alpha V_{a1}+\alpha^2V_{a2}+V_{a0}=$ $\alpha V_1+\alpha^2V_2+V_0=0$
$I_{a1}=I_{a2}=I_{ao}=I_{af}/3$ $I_1=I_2=I_0=I_{af}/3$	$I_{b1}=I_{b2}=I_{bo}= I_{bf}/3$ $\alpha^2I_{a1}=\alpha I_{a2}=I_{a0}=I_{bf}/3$ $\alpha^2I_1=\alpha I_2=I_0=I_{bf}/3$	$I_{c1}=I_{c2}=I_{co}= I_{cf}/3$ $\alpha I_{a1}=\alpha^2I_{a2}=I_{a0}=I_{cf}/3$ $\alpha I_1=\alpha^2I_2=I_0=I_{cf}/3$

In Table 1, the complex operator $\alpha=1\angle 120^\circ$.

The fault admittance method is general in the sense that any fault impedances may be represented, provided the special case of a zero impedance fault is catered for. Therefore, a line-to-ground fault on an odd phase, say on the *b* or *c* phase, poses no difficulties and is easily accommodated.

This paper presents the results of line-to-ground faults on the reference and odd phases of a simple power system obtained using the general fault admittance method.

2. Background

Sakala and Daka [1] discussed the solution procedure of the general fault admittance method. However, it is presented here in brief, showing the salient features, key equations and the solution procedure.

A single line-to-ground fault presents a low value impedance, with zero value for a direct short circuit or metallic fault, to one of the phases at the point of fault in the network. In general, a fault may be represented as shown in Figure 1.

In Figure 1, a fault at a busbar is represented by fault admittances in each phase, i.e. the inverse of the fault impedance in the phase, and the admittance in the ground path. Note that the fault admittance for a short-circuited phase is represented by an infinite value, while that for an open-circuited phase is a zero value. Thus for a line-to-ground fault the admittances Y_{bf} and Y_{cf} are zero while those for Y_{af} and Y_{gf} are infinite.

A systematic approach for using a fault admittance matrix in the general fault admittance method is given by Sakala and Daka [1]. The method is based on the work

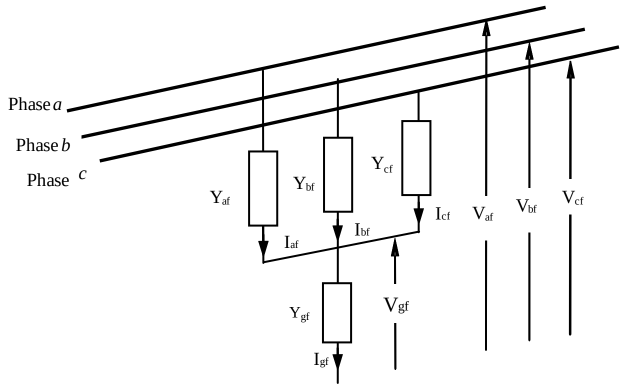

Figure 1: General Fault Representation

by Elgerd [2]. The method is summarized in this paper to give the reader a comprehensive view of the methodology.

The general fault admittance matrix is given by

$$Y_f = \left(\frac{1}{Y_{af} + Y_{bf} + Y_{cf} + Y_{gf}}\right) \times \begin{bmatrix} Y_{af}\left(Y_{bf}+Y_{cf}+Y_{gf}\right) & -Y_{af}Y_{bf} & -Y_{af}Y_{cf} \\ -Y_{af}Y_{bf} & Y_{bf}\left(Y_{af}+Y_{cf}+Y_{gf}\right) & -Y_{bf}Y_{cf} \\ -Y_{af}Y_{cf} & -Y_{bf}Y_{cf} & Y_{cf}\left(Y_{af}+Y_{bf}+Y_{gf}\right) \end{bmatrix} \quad (1)$$

Equation 1 is transformed using the symmetrical component transformation matrix be T, and its inverse be T^{-1}, where

$$T = \begin{bmatrix} 1 & 1 & 1 \\ \alpha^2 & \alpha & 1 \\ \alpha & \alpha^2 & 1 \end{bmatrix} \text{ and } T^{-1} = \frac{1}{3}\begin{bmatrix} 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \\ 1 & 1 & 1 \end{bmatrix},$$

in which $\alpha = 1\angle 120°$ is a complex operator.

The symmetrical component fault admittance matrix is given by the product

$$Y_{fs} = T^{-1} Y_f T$$

The general expression [1, 2] for Y_{fs} is given by:

$$Y_{fs} = \frac{1}{Y_{af} + Y_{bf} + Y_{cf} + Y_{gf}} \begin{bmatrix} Y_{fs11} & Y_{fs12} & Y_{fs13} \\ Y_{fs21} & Y_{fs22} & Y_{fs23} \\ Y_{fs31} & Y_{fs32} & Y_{fs33} \end{bmatrix} \quad (2)$$

where

$$Y_{fs11} = Y_{fs22} = \frac{1}{3}Y_{gf}\left(Y_{af} + Y_{bf} + Y_{cf}\right) + Y_{af}Y_{bf} + Y_{af}Y_{cf} + Y_{bf}Y_{cf} \quad Y_{fs33} = \frac{1}{3}Y_{gf}\left(Y_{af} + Y_{bf} + Y_{cf}\right)$$

$$Y_{fs12} = \frac{3}{3}Y_{gf}\left(Y_{af} + \alpha^2 Y_{bf} + \alpha Y_{cf}\right) - \left(Y_{bf}Y_{cf} + \alpha Y_{af}Y_{bf} + \alpha^2 Y_{af}Y_{cf}\right) \quad Y_{fs21} = \frac{1}{3}Y_{gf}\left(Y_{af} + \alpha Y_{bf} + \alpha^2 Y_{cf}\right) - \left(Y_{bf}Y_{cf} + \alpha^2 Y_{af}Y_{bf} + \alpha Y_{af}Y_{cf}\right)$$

$$Y_{fs13} = Y_{fs32} = \frac{1}{3}Y_{gf}\left(Y_{af} + \alpha Y_{bf} + \alpha^2 Y_{cf}\right) \text{ and}$$

$$Y_{fs31} = Y_{fs23} = \frac{1}{3}Y_{gf}\left(Y_{af} + \alpha^2 Y_{bf} + \alpha Y_{cf}\right)$$

The above expressions simplify considerably depending on the type of fault. For example, considering a balanced three-phase fault with $Y_{af} = Y_{bf} = Y_{cf} = Y$.

$$Y_{fs} = \frac{1}{3Y + Y_{gf}} \begin{bmatrix} Y\left(3Y + Y_{gf}\right) & 0 & 0 \\ 0 & Y\left(3Y + Y_{gf}\right) & 0 \\ 0 & 0 & YY_{gf} \end{bmatrix}$$

$$= \begin{bmatrix} Y & 0 & 0 \\ 0 & Y & 0 \\ 0 & 0 & \dfrac{YY_{gf}}{3Y + Y_{gf}} \end{bmatrix} \quad (3a)$$

There is no coupling between the positive, negative and zero sequence networks. Since there are no negative and zero sequence voltages before the fault there will be no corresponding currents during and after the fault.
Note that in the case that the ground is not involved, $Y_{gf} = 0$ and the symmetrical component fault admittance matrix reduces to

$$Y_{fs} = \begin{bmatrix} Y & 0 & 0 \\ 0 & Y & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (3b)$$

For a line to line fault
$Y_{af} = Y_{gf} = 0, \quad Y_{bf} = Y_{cf} = 2Y,$ i.e. $Z_{af} = Z_{gf} = \infty$

$$Y_{fs} = \frac{1}{2Y + 2Y} \begin{bmatrix} 2Y \times 2Y & -(2Y \times 2Y) & 0 \\ -(2Y \times 2Y) & 2Y \times 2Y & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= Y \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (4)$$

For a line-to-ground fault
$Y_{af} = Y, \quad Y_{bf} = Y_{cf} = 0, \quad Y_{gf} = \infty$ i.e. $Z_{gf} = 0$

$$Y_{fs} = \frac{YY_{gf}}{3(Y + Y_{gf})} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \frac{Y}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad (5)$$

Note that, although for a metallic short circuit Y is infinite the analysis is performed by means of a limit study.

2.1 Currents in the Fault

At the faulted busbar, say busbar j, the symmetrical component currents in the fault are given by:

$$I_{fsj} = Y_{fs}(U + Z_{sjj}Y_{fs})^{-1}V_{sj}^{\ 0} \quad (6)$$

where U is the unit matrix

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and Z_{sjj} is the jj^{th} component of the symmetrical component bus impedance matrix

$$Z_{sjj} = \begin{bmatrix} Z_{sjj+} & 0 & 0 \\ 0 & Z_{sjj-} & 0 \\ 0 & 0 & Z_{sjj0} \end{bmatrix}$$

The element Z_{sjj+} is the Thevenin's positive sequence impedance at the faulted busbar, Z_{sjj-} is the Thevenin's negative sequence impedance at the faulted busbar, and Z_{sjj0} is the Thevenin's zero sequence impedance at the faulted busbar.
Note that as the network is balanced the mutual terms are all zero.
In equation (6) V_{sj}^{0} is the prefault symmetrical component voltage at busbar j the faulted busbar

$$V_{sj}^{\ 0} = \begin{bmatrix} V_{sj+} \\ V_{sj-} \\ V_{sj0} \end{bmatrix} = \begin{bmatrix} V_{+} \\ 0 \\ 0 \end{bmatrix}$$

where V_{+} is the positive sequence voltage before the fault. The negative and zero sequence voltages are zero because the system is balanced prior to the fault.
The phase currents in the fault are then obtained by transformation:

$$I_{fpj} = \begin{bmatrix} I_{afj} \\ I_{bfj} \\ I_{cfj} \end{bmatrix} = TI_{fsj} \quad (7)$$

2.2 Voltages at the Busbars

The symmetrical component voltage at the faulted busbar *j* is given by:

$$V_{fsj} = \begin{bmatrix} V_{j+} \\ V_{j-} \\ V_{j0} \end{bmatrix} = \left(U + Z_{sjj}Y_{fs}\right)^{-1} V_{sj}^{\ 0} \quad (8)$$

The symmetrical component voltage at a busbar *i* for a fault at busbar *j* is given by:

$$V_{fsi} = \begin{bmatrix} V_{i+} \\ V_{i-} \\ V_{i0} \end{bmatrix} = V_{si}^{\ 0} - Z_{sij}Y_{fs}\left(U + Z_{sjj}Y_{fs}\right)^{-1} V_{sj}^{\ 0} \quad (9)$$

where $V_{si}^{\ 0} = \begin{bmatrix} V_{i+}^{\ 0} \\ 0 \\ 0 \end{bmatrix}$

gives the symmetrical component prefault voltages at busbar *i*. The negative and zero sequence prefault voltages are zero.

In equation (9), Z_{sij} gives the ij^{th} components of the symmetrical component bus impedance matrix, the mutual terms for row *i* and column *j* (corresponding to busbars *i* and *j*)

$$Z_{sij} = \begin{bmatrix} Z_{sij+} & 0 & 0 \\ 0 & Z_{sij-} & 0 \\ 0 & 0 & Z_{sij0} \end{bmatrix}$$

The phase voltages in the fault, at busbar *j*, and at busbar *i* are then obtained by transformation

$$V_{fpj} = \begin{bmatrix} V_{afi} \\ V_{bfi} \\ V_{cfi} \end{bmatrix} = TV_{fsj} \quad \text{and} \quad V_{fpi} = \begin{bmatrix} V_{afpi} \\ V_{bfpi} \\ V_{cfpi} \end{bmatrix} = TV_{fpi} \quad (10)$$

2.3 Currents in Lines and Generators

The symmetrical component currents in a line between busbars *i* and *j* is given by

$$I_{fsij} = Y_{fsij}\left(V_{fsi} - V_{fsj}\right) \quad (11)$$

where

$$Y_{fsij} = \begin{bmatrix} Y_{fsij+} & 0 & 0 \\ 0 & Y_{fsij-} & 0 \\ 0 & 0 & Y_{fsij0} \end{bmatrix}$$

is the symmetrical component admittance of the branch between busbars *i* and *j*.

The same equation applies to a generator where the source voltage will be the prefault induced voltage and the receiving end busbar voltage is the postfault voltages at the busbar.

The phase currents in the branch are found by transformation

$$I_{fpij} = \begin{bmatrix} I_{aifj} \\ I_{bifj} \\ I_{cfij} \end{bmatrix} = TI_{fsij} \quad (12)$$

3. Line-to-Ground Fault Simulation

Equation (5) gives the symmetrical component fault admittance matrix for a line-to-ground fault. It is restated here for easy of reference:

$$Y_{fs} = \frac{YY_{gf}}{3\left(Y + Y_{gf}\right)}\begin{bmatrix}1 & 1 & 1\\1 & 1 & 1\\1 & 1 & 1\end{bmatrix} = \frac{Y}{3}\begin{bmatrix}1 & 1 & 1\\1 & 1 & 1\\1 & 1 & 1\end{bmatrix}$$

The value Y is the fault admittance in the faulted phase with the ground assumed to be a metallic contact, with Z_{gf} = 0.

The symmetrical component fault admittance matrix may be substituted in equation (6) which to obtain a simplified value of I_{fsj} given in equation (6a), in which V_j^0 is the prefault voltage on bus bar j. The simplified formulation in equation (13) is useful for checking the accuracy of the symmetrical component currents in the fault when the general form is used.

$$I_{fsj} = V_j^0 \frac{\frac{Y}{3}}{1 + \left(\frac{Y}{3}\right)\left(Z_{sjj+} + Z_{sjj-} + Z_{sjj0}\right)}\begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$= V_j^0 \frac{1}{\left(\frac{3}{Y}\right) + \left(Z_{sjj+} + Z_{sjj-} + Z_{sjj0}\right)}\begin{bmatrix}1\\1\\1\end{bmatrix} \qquad (13)$$

The impedances required to simulate the line-to-ground fault in general terms are the impedance in the faulted phase and the ground path. The impedances in the faulted phase and ground are assumed equal to 5×10^{-10} Ω. In practice, this is not significant as the two values are added to arrive at the total impedance in the fault. The open circuited phases are on open circuit, which is simulated by a very high resistance of the order of 10^{50} Ω.

4. Computation of the Line-to-Ground Faults on Reference and Odd Phases

A computer program has been developed, based on the equations (1) to (13), to solve unbalanced faults for a general power system using the fault admittance matrix method. The program is applied on a simple power system comprising of three bus bars to solve for line-to-ground faults on the three phases. A simple system is chosen because it is easy to check the results against those that are obtained by hand.

4.1 Sample System

Figure 2 shows a simple three bus bar power system with one generator, one transformer and one transmission line. The system if configured based on the simple power system that Saadat uses [3].

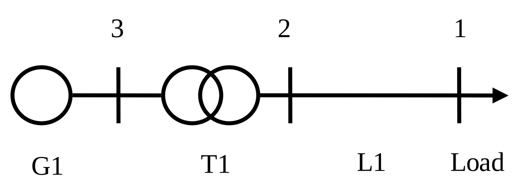

Figure 2: Sample Three Bus Bar System

The power system per unit data is given in Table 1, where the subscripts *1*, *2*, and *0* refer to the positive, negative and zero sequence values respectively. The neutral point of the generator is grounded through a zero impedance. The transformer windings are delta connected on the low voltage side and earthed-star connected on the high voltage side, with the neutral solidly grounded. The phase shift of the transformer is 30°, i.e. from the generator side to the line side. Figure 3 shows the relationship of transformer voltages for a delta star transformer connection Yd11 that has a 30° phase shift.

Table 1: Power System Data

Item	S_{base} (MVA)	V_{base} (kV)	X_1 (pu)	X_2 (pu)	X_0 (pu)
G_1	100	20	0.15	0.15	0.05
T_1	100	20/220	0.1	0.1	0.1
L_1	100	220	0.25	0.25	0.7125

The computer program incorporates an input program that calculates the sequence admittance and impedance matrices and then assembles the symmetrical component bus impedance matrix for the power system. The symmetrical component bus impedance incorporates all the sequences values and has 3*n* rows and 3*n* columns where *n* is the number of bus bars. In general, the mutual terms between sequence values are zero as a three-phase power system is, by design, balanced.

The power system is assumed to be at no load before the occurrence of a fault. In practice the pre-fault conditions, established by a load flow study may be used. In developing a computer program the assumption of no load, and therefore voltages of *1.0* per unit at the bus bars and in the generator, is adequate.

The line-to-ground faults are assumed to be at busbar *1*, the load busbar. They are described by the impedances in the respective phases and in the ground path.

The presence of the delta-earthed-star transformer poses a challenge in terms of its modelling. In the computer program, the transformer is considered as a normal star-star connection, for the positive and negative sequence networks. The phase shifts are incorporated when assembling the sequence currents to obtain the phase values. In particular on the delta connected side of the transformer the positive sequence currents' angles are increased by the phase shift while the angle of the negative sequence currents are reduced by the same value. Allowance is made in the phase current for the line current factor in a delta star transformer. The zero sequence currents, if any, are not affected by the phase shifts.

5. Results and Discussions

5.1 Fault simulation impedances

The Thevenin's self-sequence impedances of the network seen from the faulted bus bar are:

$$\begin{bmatrix} j0.5 & 0 & 0 \\ 0 & j0.5 & 0 \\ 0 & 0 & j0.8125 \end{bmatrix}$$

In the classical solution, the sequence currents due to a line-to-ground fault on phase *a* are equal and are found by inverting the sum of the diagonal elements. Thus the sequence (positive, negative and zero) currents due to fault in the reference phase at the faulted bus bar are:

$$-j\begin{bmatrix} 0.5517 \\ 0.5517 \\ 0.5517 \end{bmatrix}$$

These sequence currents are compared with computed ones for the different line-to-ground faults, in the reference phase *a* and the odd phases *b* and *c*.

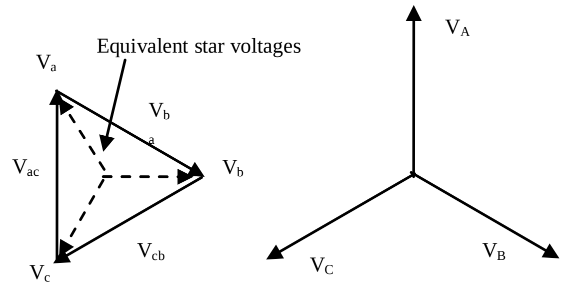

Figure 3: Delta-star Transformer Voltages for Yd11

5.2 Simulation Results

The results obtained from the computer program are listed in Table 3. A summary of the transformer phase currents is given in Figures 4a, 4b and 4c for single line-to-ground faults in phases *a*, *b* and *c* respectively.

5.3 Fault Admittance Matrix and Sequence Impedances at the Faulted Busbar.

The symmetrical component fault admittance matrix obtained from the program for the line-to-ground faults are in agreement with the theoretical values, obtained using equation (5). The self-sequence impedances at the faulted bus bar obtained from the program are equal to the theoretical values.

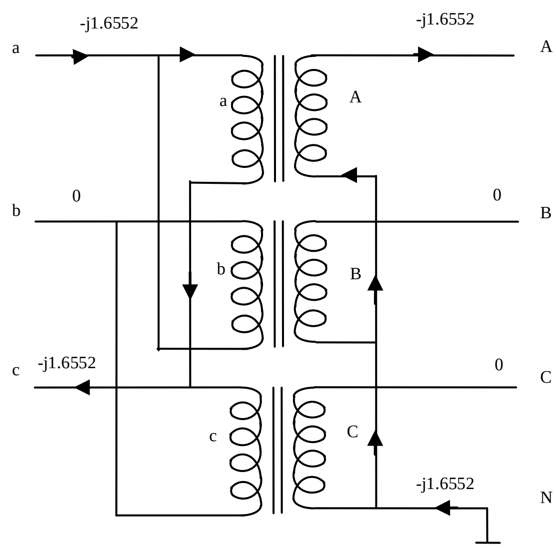

Figure 4a: Transformer currents for a line-to-ground fault in phase *a*.

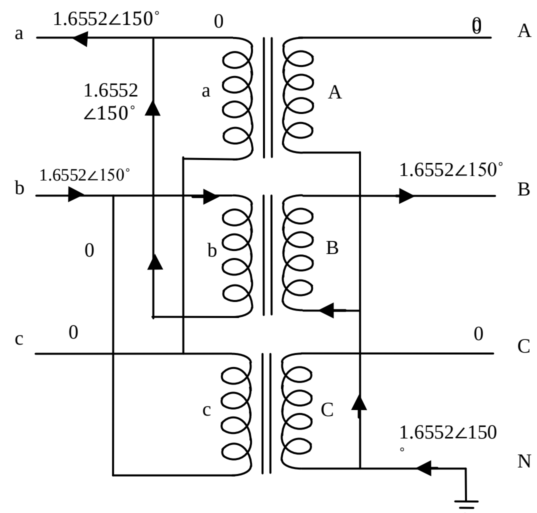

Figure 4b: Transformer currents for a line-to-ground fault in phase *b*.

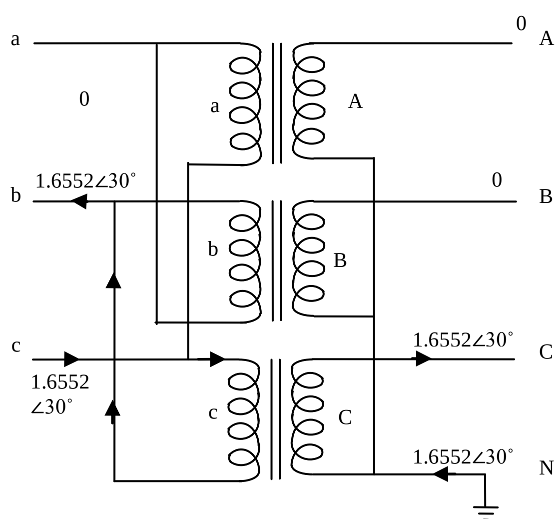

Figure 4c: Transformer currents for a line-to-ground fault in phase *c*

5.4 Fault Currents

The symmetrical component fault currents obtained from the program using equations (6) and (13) are in agreement with the theoretical values. In particular, the sequence currents for the line-to-ground fault in the reference phase *a* are equal to each other. This is consistent with the classical approach that connects the sequence networks in series.

When the line-to-ground fault is on the odd phase *b* the sequence currents are no longer equal in that although the magnitudes are the same their phase angles are different. The negative sequence component of the phase current in the *b* phase leads the respective component in the *a* phase by 120° while the zero sequence component for the fault in the *b* phase leads the zero sequence component in the a phase by 240°.

The results are consistent with the requirement that for a line-to-ground fault in phase *b* the current symmetrical component constraints are met, that is $\alpha^2 I_1 = \alpha I_2 = I_0 = \frac{I_{bf}}{3}$.

Similarly, when the line-to-ground fault is on odd phase c the negative and zero sequence component currents lead the positive sequence current by *240˚* and *120˚* respectively. The symmetrical component sequence currents constraint is met; i.e. $\alpha I_1 = \alpha^2 I_2 = I_0 = \frac{I_{cf}}{3}$. and that $I_{b1} = I_{b2} = I_{b0}$ which is $\alpha^2 I_1 = \alpha I_2 = I_0$, where *α=1∠120˚*, as earlier defined.

The phase currents in the fault obtained from the program are in agreement with the theoretical values. In particular, the currents in the healthy phases are zero and the currents in the faulted phase lag the voltages by *90º*, since the resistances in the networks are zero.

The phase currents in the transmission line are equal to the currents in the faults. Note that the current at the receiving end of the line is by convention considered as flowing into the line, rather than out of it.

Figures 4a, 4b and 4c summarise the transformer phase currents for the line-to-ground fault on phases *a*, *b* and *c* respectively. The currents in the transformer, on the line side, are equal to the currents in the line, after allowing for the sign changes due to convention. Note that the fault current only flows in the winding of the faulted phase on the earthed-star connected side. The currents at the sending end of the transformer, the delta connected side flow into the phase *a* and phase *c*, phases *b* and *a*, and phases *c* and *b* terminals of the transformer for line-to-ground faults on phases *a*, *b* and *c* respectively. In all the cases only the transformer windings in the faulted phase carry the fault current. The other two windings on the delta-connected side do not carry currents as the corresponding windings on the earthed-star connected side have no current. These results satisfy the ampere-turn balance requirements of the transformer.

The phase fault currents flowing from the generator are equal to the phase currents into the transformer for all the three line-to-ground faults. For each fault phase fault currents only flow in two phases of the generator; *a* and phase *c*, phases *b* and *a*, and phases *c* and *b* for line-to-ground faults on phases *a*, *b* and *c* respectively. It is a feature of the delta earthed-star connection that a single-phase load on the star side is supplied from two phases on the delta side.

5.5 Fault Voltages

The symmetrical component voltages at the fault point obtained from the program using equation (9) are in agreement with the theoretical values. In particular, the sequence voltages constraints in Table 1 are satisfied; The sequence component voltages phase summate to zero, for the line-to-ground fault on the reference consistent with the concept of the networks being connected in series. When the line-to-ground fault is on the odd phases the respective symmetrical component voltage constraints are also satisfied; i.e. $V_{b1} + V_{b2} + V_{b0} = 0$ that is $\alpha^2 V_1 + \alpha V_2 + V_0 = 0$ for the line-to-ground fault on odd phase *b* and $V_{c1} + V_{c2} + V_{c0} = 0$ and that $\alpha V_1 + \alpha^2 V_2 + V_0 = 0$ for the line-to-ground fault on odd phase *c*.

Also note that the phase voltages of the faulted phases are zero while the voltages in the healthy phases are of equal magnitude, and greater than unity, the rated value.

The phase voltages at bus bar 2 show that the voltages in the faulted phases are *67%* of the prefault values while the voltages in the healthy phases are *96%* of the prefault values. At bus bar 3, the voltages lead the voltages at bus bar 2. In particular, the voltages in the faulted phases leads the corresponding phase voltages at bus bar 2 by *34.7˚*. The increase in the phase shift between phase voltages of the faulted phases is due to the voltage drop in the transformer. The voltages of the phases which do not carry any fault currents on the delta connected side of the transformer, i.e. phase *b*, *c* and *a* for line-to-ground faults on phases *a*, *b* and *c* respectively, are equal to their pre-fault values. Furthermore, the voltages of the phases that do not carry any current at bus bar 3 lead those of bus bar 2 by *30˚*. This is as expected since there are no current in the respective phases of the generator. The voltages of the phases that carry the return currents at bus bar 3 leads those of the respective phases at bus bar 2 by *29.6˚*.

6. Conclusions

The general fault admittance method may be used to study line-to-ground faults on the reference phase as well as on the odd phases. The ability to handle line-to-ground faults on odd phases makes it easier to study these faults, since the method does not require the knowledge needed to translate the fault from the reference phase to the odd phase.

The line-to-ground fault is interesting for studying the delta-earthed-star transformer arrangement. It is seen that although only one phase carries the fault current on the earthed-star side the currents on the delta- connected side are in two phases. Phase shifts in the transformer can be deduced from the results. The results give an insight in the effect that a delta-earthed-star transformer has on a power system during line-to-ground faults.

The main advantage of the general fault admittance method is that the user is not required to know before hand how the sequence networks should be connected at the fault point in order to obtain the sequence fault currents and voltages. The user can deduce the various relationships from the results. The method is therefore easier to use and teach than the classical approach in which each network is solved in isolation and then the results combined to obtain initially the sequence currents and voltages and then the phase quantities.

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Table 3: Simulation Results - Unbalanced Fault Study

General Fault Admittance Method - Delta-star Transformer Model
Number of busbars = 3
Number of transmission lines = 1
Number of transformers = 1
Number of generators = 1
Faulted busbar = 1
Fault type = 4

General Line to line to line-to-ground fault
Fault impedances

		Phase A fault
Phase a	(R + jX)	5.0000e-010 +j 0.0000e+000
Phase b	(R + jX)	1.0000e+050 +j 0.0000e+000
Phase c	(R + jX)	1.0000e+050 +j 0.0000e+000
Ground	(R + jX)	5.0000e-010 +j 0.0000e+000
		Phase B fault
Phase a	(R + jX)	1.0000e+050 +j 0.0000e+000
Phase b	(R + jX)	5.0000e-010 +j 0.0000e+000
Phase c	(R + jX)	1.0000e+050 +j 0.0000e+000
Ground	(R + jX)	5.0000e-010 +j 0.0000e+000
		Phase C fault
Phase a	(R + jX)	1.0000e+050 +j 0.0000e+000
Phase b	(R + jX)	1.0000e+050 +j 0.0000e+000
Phase c	(R + jX)	5.0000e-010 +j 0.0000e+000
Ground	(R + jX)	5.0000e-010 +j 0.0000e+000

Fault Admittance Matrix
Real and imaginary parts of Fault Admittance Matrix

	Phase A fault	
3.3333e+008 +j 0.0000e+000	3.3333e+008 +j 0.0000e+000	3.3333e+008 +j 0.0000e+000
3.3333e+008 +j 0.0000e+000	3.3333e+008 +j 0.0000e+000	3.3333e+008 +j 0.0000e+000
3.3333e+008 +j 0.0000e+000	3.3333e+008 +j 0.0000e+000	3.3333e+008 +j 0.0000e+000
	Phase B fault	
3.3333e+008 +j 0.0000e+000	-1.6667e+008 +j -2.8868e+008	-1.6667e+008 +j 2.8868e+008
-1.6667e+008 +j 2.8868e+008	3.3333e+008 +j 0.0000e+000	-1.6667e+008 +j -2.8868e+008
-1.6667e+008 +j -2.8868e+008	-1.6667e+008 +j 2.8868e+008	3.3333e+008 +j 0.0000e+000
	Phase C fault	
3.3333e+008 +j 0.0000e+000	-1.6667e+008 +j 2.8868e+008	-1.6667e+008 +j -2.8868e+008
-1.6667e+008 +j -2.8868e+008	3.3333e+008 +j 0.0000e+000	-1.6667e+008 +j 2.8868e+008
-1.6667e+008 +j 2.8868e+008	-1.6667e+008 +j -2.8868e+008	3.3333e+008 +j 0.0000e+000

Thevenin's Symmetrical Component Impedance Matrix of Faulted Busbar
Real and imaginary parts of Symmetrical Component Impedance Matrix

0.0000 +j 0.5000	0.0000 +j 0.0000	0.0000 +j 0.0000
0.0000 +j 0.0000	0.0000 +j 0.5000	0.0000 +j 0.0000
0.0000 +j 0.0000	0.0000 +j 0.0000	0.0000 +j 0.8125

Fault current in Symmetrical Components
Real and imaginary parts

	Phase A fault	Phase B fault	Phase C
+ve	0.0000 +j -0.5517	0.0000 +j -0.5517	0.0000 +j -0.5517
-ve	0.0000 +j -0.5517	0.4778 +j 0.2759	-0.4778 +j 0.2759
zero	0.0000 +j -0.5517	-0.4778 +j 0.2759	0.4778 +j 0.2759

Magnitude and angle

	Phase A fault		Phase B fault		Phase C fault	
	magn	angle	magn	angle	magn	angle
+ve	0.5517	-90.0000	0.5517	-90.0000	0.5517	-90.0000
-ve	0.5517	-90.0000	0.5517	30.0000	0.5517	150.0000
zero	0.5517	-90.0000	0.5517	150.0000	0.5517	30.0000

Fault current in phase components

In Rectangular and Polar Coordinates

Phase A fault

	real		imag	magn	angle
Phase a	0.0000	+j	-1.6552	1.6552	-90.0000
Phase b	0.0000	+j	0.0000	0.0000	0.0000
Phase c	0.0000	+j	0.0000	0.0000	0.0000

Phase B fault

	real		imag	magn	angle
Phase a	-0.0000	+j	-0.0000	0.0000	201.8952
Phase b	-1.4334	+j	0.8276	1.6552	150.0000
Phase c	0.0000	+j	-0.0000	0.0000	-46.0481

Phase C fault

	real		imag	magn	angle
Phase a	0.0000	+j	-0.0000	0.0000	-10.1831
Phase b	0.0000	+j	0.0000	0.0000	21.5591
Phase c	1.4334	+j	0.8276	1.6552	30.0000

Symmetrical Component Voltages at Faulted Busbar

In Rectangular and Polar Coordinates

Phase A fault

	real	imag	magn	angle
+ve	0.7241	-0.0000	0.7241	-0.0000
-ve	-0.2759	-0.0000	0.2759	180.0000
zero	-0.4483	-0.0000	0.4483	180.0000

Phase B fault

	real	imag	magn	angle
+ve	0.7241	0.0000	0.7241	0.0000
-ve	0.1379	-0.2389	0.2759	-60.0000
zero	0.2241	0.3882	0.4483	60.0000

Phase C fault

	real	imag	magn	angle
+ve	0.7241	-0.0000	0.7241	-0.0000
-ve	0.1379	0.2389	0.2759	60.0000
zero	0.2241	-0.3882	0.4483	-60.0000

Phase Voltages at Faulted Busbar

Phase A fault

	real	imag	magn	angle
Phase a	0.0000	-0.0000	0.0000	-90.0000
Phase b	-0.6724	-0.8660	1.0964	232.1729
Phase c	-0.6724	0.8660	1.0964	127.8271

Phase B fault

	real	imag	magn	angle
Phase a	1.0862	0.1493	1.0964	7.8271
Phase b	-0.0000	0.0000	0.0000	163.5968
Phase c	-0.4138	1.0153	1.0964	112.1729

Phase C fault

	real	imag	magn	angle
Phase a	1.0862	-0.1493	1.0964	-7.8271
Phase b	-0.4138	-1.0153	1.0964	247.8271
Phase c	0.0000	-0.0000	0.0000	-4.7935

Postfault Voltages at Busbar number = 1

Phase A fault

	real	imag	magn	angle
Phase a	0.0000	-0.0000	0.0000	-90.0000
Phase b	-0.6724	-0.8660	1.0964	232.1729
Phase c	-0.6724	0.8660	1.0964	127.8271

Phase B fault

	real	imag	magn	angle
Phase a	1.0862	0.1493	1.0964	7.8271
Phase b	-0.0000	0.0000	0.0000	163.5968
Phase c	-0.4138	1.0153	1.0964	112.1729

Phase C fault

	real	imag	magn	angle
Phase a	1.0862	-0.1493	1.0964	-7.8271
Phase b	-0.4138	-1.0153	1.0964	247.8271
Phase c	0.0000	-0.0000	0.0000	-4.7935

Postfault Voltages at Busbar number = 2

Phase A fault

	real	imag	magn	angle
Phase a	0.6690	-0.0000	0.6690	-0.0000
Phase b	-0.4172	-0.8660	0.9613	244.2758
Phase c	-0.4172	0.8660	0.9613	115.7242

Phase B fault

	real	imag	magn	angle
Phase a	0.9586	-0.0717	0.9613	-4.2758
Phase b	-0.3345	-0.5793	0.6690	240.0000
Phase c	-0.5414	0.7944	0.9613	124.2758

Phase C fault

	real	imag	magn	angle
Phase a	0.9586	0.0717	0.9613	4.2758
Phase b	-0.5414	-0.7944	0.9613	235.7242
Phase c	-0.3345	0.5793	0.6690	120.0000

Postfault Voltages at Busbar number = 3

Phase A fault

	real	imag	magn	angle
Phase a	0.7227	0.5000	0.8788	34.6780
Phase b	0.0000	-1.0000	1.0000	-90.0000
Phase c	-0.7227	0.5000	0.8788	145.3220

Phase B fault

	real	imag	magn	angle
Phase a	0.7944	0.3759	0.8788	25.3220
Phase b	0.0717	-0.8759	0.8788	-85.3220
Phase c	-0.8660	0.5000	1.0000	150.0000

Phase C fault

	real	imag	magn	angle
Phase a	0.8660	0.5000	1.0000	30.0000
Phase b	-0.0717	-0.8759	0.8788	265.3220
Phase c	-0.7944	0.3759	0.8788	154.6780

Postfault Currents in Lines

Phase A fault

			Phase a		Phase b		Phase c	
Line No.	SE Bus	RE Bus	Current Magn	Current Angle Deg.	Current Magn	Current Angle Deg.	Current Magn	Current Angle Deg.
1	2	1	1.6552	-90.0000	0.0000	-21.8014	0.0000	201.8014
1	1	2	1.6552	90.0000	0.0000	158.1986	0.0000	21.8014

Phase B fault

Line No.	SE Bus	RE Bus	Phase a Current Magn	Phase a Current Angle Deg.	Phase b Current Magn	Phase b Current Angle Deg.	Phase c Current Magn	Phase c Current Angle Deg.
1	2	1	0.0000	201.8952	1.6552	150.0000	0.0000	-46.0481
1	1	2	0.0000	21.8952	1.6552	-30.0000	0.0000	133.9519

Phase C fault

Line No.	SE Bus	RE Bus	Phase a Current Magn	Phase a Current Angle Deg.	Phase b Current Magn	Phase b Current Angle Deg.	Phase c Current Magn	Phase c Current Angle Deg.
1	2	1	0.0000	-10.1831	0.0000	21.5591	1.6552	30.0000
1	1	2	0.0000	169.8169	0.0000	201.5591	1.6552	210.0000

Postfault Currents in Transformers

Phase A fault

Transf No.	SE Bus	RE Bus	Phase a Current Magn	Phase a Current Angle Deg.	Phase b Current Magn	Phase b Current Angle Deg.	Phase c Current Magn	Phase c Current Angle Deg.
1	3	2	1.6552	-90.0000	0.0000	-27.1870	1.6552	90.0000
1	2	3	1.6552	90.0000	0.0000	-2.5641	0.0000	241.2348

Phase B fault

Transf No.	SE Bus	RE Bus	Phase a Current Magn	Phase a Current Angle Deg.	Phase b Current Magn	Phase b Current Angle Deg.	Phase c Current Magn	Phase c Current Angle Deg.
1	3	2	1.6552	-30.0000	1.6552	150.0000	0.0000	218.4746
1	2	3	0.0000	117.2291	1.6552	-30.0000	0.0000	229.0699

Phase C fault

Transf No.	SE Bus	RE Bus	Phase a Current Magn	Phase a Current Angle Deg.	Phase b Current Magn	Phase b Current Angle Deg.	Phase c Current Magn	Phase c Current Angle Deg.
1	3	2	0.0000	90.3204	1.6552	210.0000	1.6552	30.0000
1	2	3	0.0000	121.8244	0.0000	0.6431	1.6552	210.0000

Postfault Currents in Generators

Phase A fault

Gen No.	SE Bus	RE Bus	Phase a Current Magn	Phase a Current Angle Deg.	Phase b Current Magn	Phase b Current Angle Deg.	Phase c Current Magn	Phase c Current Angle Deg.
1	4	3	1.6552	-90.0000	0.0000	90.0000	1.6552	90.0000

Phase B fault

Gen No.	SE Bus	RE Bus	Phase a Current Magn	Phase a Current Angle Deg.	Phase b Current Magn	Phase b Current Angle Deg.	Phase c Current Magn	Phase c Current Angle Deg.
1	4	3	1.6552	-30.0000	1.6552	150.0000	0.0000	-26.5513

Phase C fault

Gen No.	SE Bus	RE Bus	Phase a Current Magn	Phase a Current Angle Deg.	Phase b Current Magn	Phase b Current Angle Deg.	Phase c Current Magn	Phase c Current Angle Deg.
1	4	3	0.0000	263.1498	1.6552	210.0000	1.6552	30.0000