The Entanglement Entropy of Some Few-qubit XY Model in a Transverse Field

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Abstract

The entanglement entropy of three-qubit and four-qubit for anisotropic XY model in a transverse field are discussed in this paper. We find that the entanglement entropy scales with the volume of the subsystem with the increment of temperature, and the RS *(the rate of the entropies of the subsystems) changes from some constant to another one with temperature getting high, and has the similar behavior with the transverse field varies. The* ΔS *(the entropy of the system minus the sum of the entropies of the subsystems) from zero to some negative constant with the field strengthen, and the derivative of which can be used to study the quantum phase transition.*

Keywords: entanglement entropy, anisotropic XY model, few-qubit, quantum phase transition.

1. Introduction

Today, the quantum computation has been a subject of very active research [Robert P,2007, Amin M H S,,2008], and in quantum information, there has many interesting features which have no classical analogue. One of them is entanglement or quantum correlations. [Deutsch D, 1985, Lloyd S, 1996] It has been the object of intensive study for last years because it is the entanglement that makes possible to develop effective algorithms solving many tasks in computing, communication, and cryptography.

How can we define a measure of entanglement? As we now have agreed that the entropy of entanglement is a good measure of entanglement for pure states, and the entanglement of formation is the first measure of entanglement for mixed states and whose definition is based on the entropy of entanglement. So the entanglement play the key role in the quantum information. The precise statement says that an average random state in the Hilbert space is known to carry maximal von Neumann entropy. Let us describe in more detail this point. Consider a partition of the original state into two parties, A and B. If party A ignores party B, the description of its subsystem is based on the reduced density matrix

$$\rho_A = tr_B \,|\,\varphi><\varphi\,| \tag{1}$$

The description that party A is making of the system ignores quantum correlations between A and B. If A would suddenly discover that it was correlated to B a surprise would take place. The amount of that surprise is quantified by the von Neumann entropy

$$S(\rho_A) = -tr(\rho_A \ln \rho_A) \tag{2}$$

We know that it for a pure state the entropies for party A and B will find the same result. But for mixed state, How the entanglement entropy behaves? About the entanglement entropy, Nicolas Laflorencie et al study the boundary effects in the critical scaling of entanglement entropy in 1D systems [Laflorencie N,2006], and Jose I. Latorre discuss the relation between entanglement entropy and the computational difficulty of classically simulating Quantum Mechanics [Latorre J I, 2007].

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In this paper we will discuss the entanglement entropy of few qubits for anisotropic XY model in a transverse field. We first write the Hamiltonian of the system in the standard basis, and to calculate the entanglement entropy, and find the entropy behaves with the temperature, transverse field strength and the anisotropic parameter. We find the entanglement entropy scales with the volume of the subsystem with the increment of temperature, and the RS (the ratio of the entropies of the subsystems) changes from some constant to another one with temperature getting high, and has the similar behavior with the transverse field varies. In the last part we discuss the quantum phase transition by use the derivative of ΔS .

2. The model and the method

The Hamiltonian for the anisotropic XY model is given by

$$H = -\sum_{i=1}^{L-1}\{\frac{\lambda}{2}[(1+\gamma)\sigma_i^x\sigma_{i+1}^x + (1-\gamma)\sigma_i^y\sigma_{i+1}^y] + \sigma_i^z\} \tag{3}$$

Where λ_i are the nearest neighbor interactions which are relative to external transverse field, σ_i^α is the α th Pauli matrix($\alpha = x, y\ or\ z$)on site i, L is the number of sites(in our paper we choose $L=3$ and $L=4$), and γ is the degree of anisotropy.

For three-qubit system, that is $L=3$, in the standard basis, $\{|000>,|001>,|010>,|011>,|100>,|101>,|110>,|111>\}$, the Hamiltonian can be written

$$\begin{vmatrix} 3 & 0 & 0 & -\lambda\gamma & 0 & 0 & -\lambda\gamma & 0 \\ 0 & 1 & -\lambda & 0 & 0 & 0 & 0 & -\lambda\gamma \\ 0 & -\lambda & 1 & 0 & -\lambda & 0 & 0 & 0 \\ -\lambda\gamma & 0 & 0 & -1 & 0 & -\lambda & 0 & 0 \\ 0 & 0 & -\lambda & 0 & 1 & 0 & 0 & -\lambda\gamma \\ 0 & 0 & 0 & -\lambda & 0 & -1 & -\lambda & 0 \\ -\lambda\gamma & 0 & 0 & 0 & 0 & -\lambda & -1 & 0 \\ 0 & -\lambda\gamma & 0 & 0 & -\lambda\gamma & 0 & 0 & -3 \end{vmatrix} \tag{4}$$

Similarly, for the four-qubit system, the standard basis is $\{|0000>,|0001>,...,|1110>,|1111>\}$, and we can write the Hamiltonian which is 16×16 matrix.The state of the system at thermal equilibrium is

$$\rho(T) = \frac{\exp(-\frac{H}{kT})}{Z}, Z = tr(\exp(-\frac{H}{kT})) \tag{5}$$

Here, k is the Boltzman's constant. In this letter, we choose $k=1$. Then we can calculate the reduced density matrix for part A and part B, and we can study the entropy $S(\rho)$ (S), $S(\rho_A)$ (S_A) and $S(\rho_B)$ (S_B). With changing of the parameters, such as temperature, anisotropy and transverse field, we can discuss the behavior of the entropies.

3. Entropies of the systems

We study the entropies of the three- and four-spin systems. For spin-three system, we choose the near two spins as part A, the last spin as part B; for the spin-four system we choose the near three spins as part A and the last one as part B.

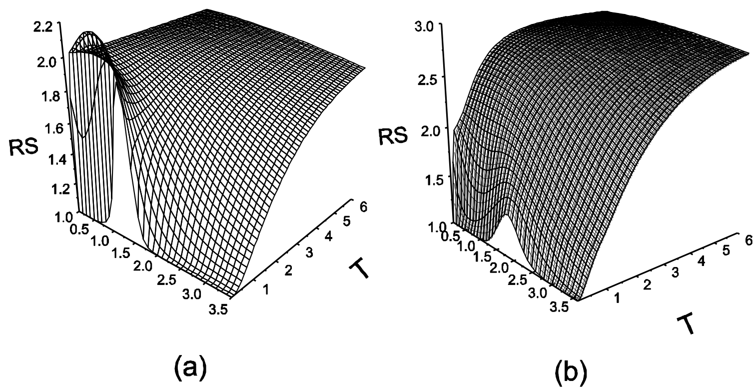

Figure 1. ***The ratio of the entropy S_A and S_B with the temperature and the interaction λ. The graph (a) and (b) correspond to the system of spin-three and spin-four, respectively.***

From Fig.1, we can easily find that the rate of S_A and S_B, For spin-three system, we can get that the ratio of the entropies tends from constant 1 to the constant 2 with the temperature increases, and for spin-four one, the rate varies from 1 to 3. At zero temperature, it has been shown that the entanglement entropies of the part system A and B are the same. It is easily to understand that with the temperature becomes higher, the quantum entanglement becomes to zero, and the system comes to the classic system, and the interaction can be negative relative to the temperature, so the entropy will scale with the size of the system. At some high temperature, Fig.1 shows that the rate of the entropy from the constant 2 or 3 to 1 with nearest neighbor interaction λ bigger, that is the transverse field strength smaller. When the transverse field strengthened to very high value, the quantum states will be destroyed to classic state, when the interaction is negative, the entropy scales with the size of the system. If the the parameter λ increase, that is the field turns slender，the state becomes gradually to a pure state, and the entropies of the subsystem will be the same.Fig.2 shows the entropy of the system and the part systems at some anisotropic parameter and some temperature. From the figure, we see that the entropy of the system and the part A changes from low to high and then to low again, it is due to the competition of the anisotropy and the nearest neighbor interaction. From Fig.2, we can also know that, the entropy proportionates to the size of the spin-system, for spin-three system the ratio of S, S_A, and S_B is $3:2:1$, and $4:3:1$ for spin-four one, when the interaction λ tends to zero. And with the interaction becomes strong, the entropy of the whole system changes to zero gradually, and the values of S_A is same to that of S_B. It is because that with the strengthening of the interaction, that is the transverse field turns from very strong to zero, the mixed state of the system will become to pure state.

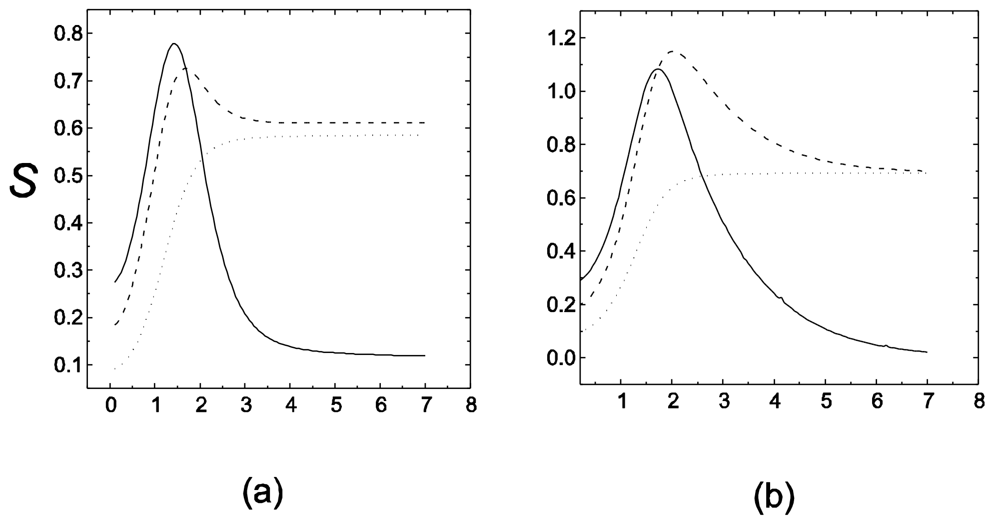

Figure 2. ***The entropy*** S ***,*** S_A ***and*** S_B ***with the nearest neighbor interaction. The graph (a) and (b) correspond to the system of spin-three and spin-four. The solid, dash and dot lines correspond to*** S ***,*** S_A ***and*** S_B***, respectively. Where*** $T = 0.5$ ***and*** $\gamma = 0.2$

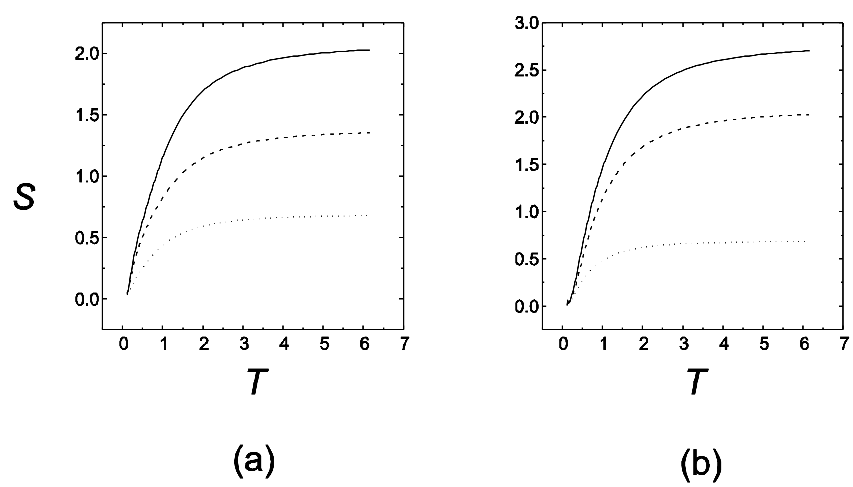

Figure 3. ***The entropy*** S ***,*** S_A ***and*** S_B ***with the temperature. The graph (a) and (b) correspond to the system of spin-three and spin-four. The solid, dash and dot lines correspond to*** S ***,*** S_A ***and*** S_B***, respectively. Where*** $\lambda = 1.0$ ***and*** $\gamma = 0.2$

With the method of the above,we can discuss the entanglement entropy of the system with temperature at some λ and γ, as shown in Fig.3. From the figure, at low temperature, the entropy S_A and S_B are equal, and with the temperature becomes high, they are proportional to the size of the spin system.

4. Discussion and conclusion

We know that the derivatives of nearest-neighbor concurrence diverge at quantum critical points, and can study the transition with the concurrence. [Zhang L F,2005] Similarly in our model, we can consume that the variable ΔS can be used to study the transition of the spin chain.

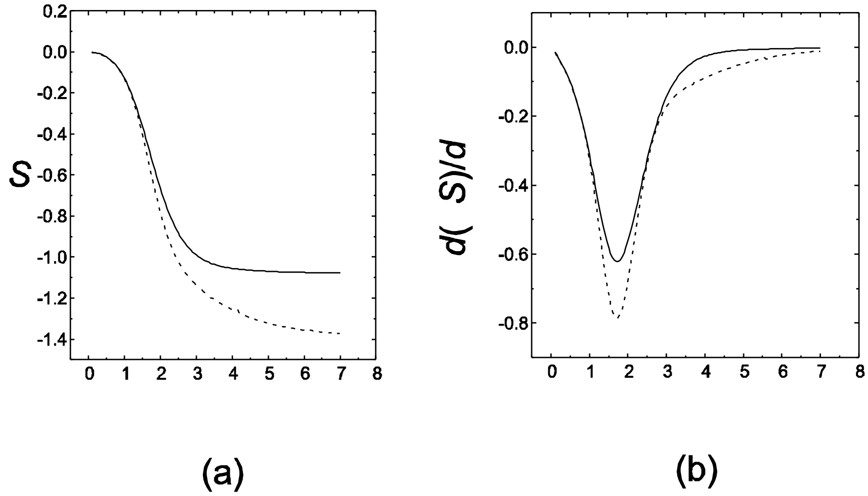

Figure 4. ***The*** ΔS ***changes with parameter*** λ ***(the left graph(a)), the right graph is the curve*** $\frac{d(\Delta S)}{d\lambda}$ ***on increasing of*** λ***. The solid and dash lines correspond to spin-three system and spin-four one. Where Where*** $T = 0.5$ ***and*** $\gamma = 0.2$***.***

From Fig.4, we can see that on increasing the size of the chain, the ΔS changes(see the left figure(a)), and the minimum of $\frac{d(\Delta S)}{d\lambda}$ changes and the curve of $\frac{d(\Delta S)}{d\lambda}$ diverges at the critical λ_c. The λ_m (tending to λ_c with the size of the spin-chain increasing) changes with the temperature, and we picture the λ_m with parameter T, at some $\gamma = 0.2$, for spin-three system(see Fig.5). From Fig.5, we can see that the critical λ is proportional to temperature. In this paper,we study the entanglement entropy of three-qubit and four-qubit for anisotropic XY model in a transverse field. We find that the entanglement entropy scales with the volume of the subsystem with the increment of temperature, and the RS (the rate of the entropies of the subsystems) changes from some constant to another one with temperature getting high, and has the similar behavior with the transverse field varies. The ΔS (the entropy of the system minus the sum of the entropies of the subsystems) from zero to some negative constant with the field strengthen, and the derivative of which can be used to study the quantum phase transition. The curve of $\frac{d(\Delta S)}{d\lambda}$ diverges at the critical λ_c. The λ_m is proportional to temperature.

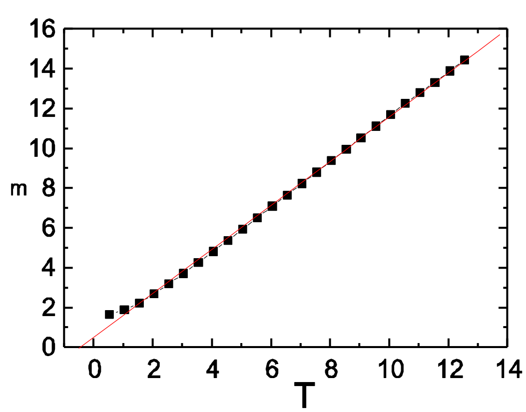

Figure 5. ***The*** λ_m ***(tending to*** λ_c***, at which*** $\frac{d(\Delta S)}{d\lambda}$ ***diverges, with the size of the spin-chain increasing) increasing with parameter*** T***, at*** $\gamma = 0.2$***, for spin-three system.***

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