On Tangles and Braids

M.El-Ghoul*

H.El-Zohny**

Z.M.Hagrass**

Abstract

We will introduce the methods by which we change the tangles into braids. We will explain these changes to convert tangles to braids. The matrices representing these changes will be discussed.

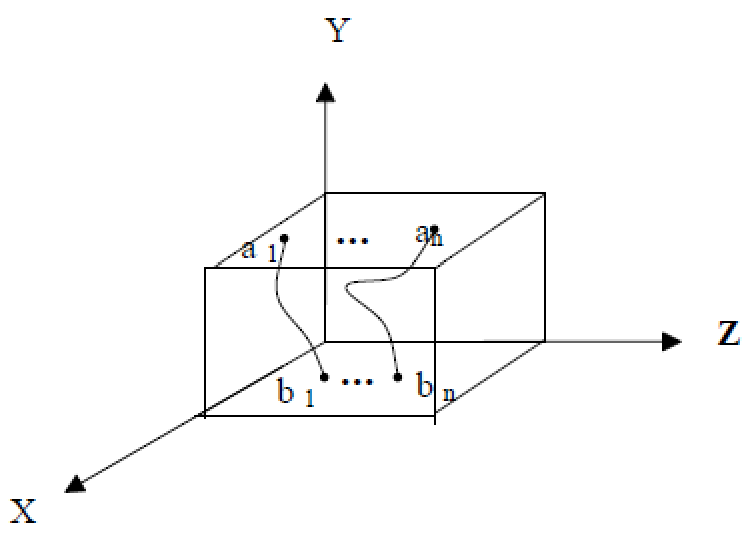
Keyword: braids, tangles

Introduction

Conway developed tangle theory and invented a system of notation for tabulating knots, nowadays known as Conway notation. Tangle theory can be considered analogous to knot theory except instead of closed loop we use string whose end are nailed down. Tangles have been shown to be useful in studying DNA topology. Braid groups were introduced explicitly by Emil Artin in 1925, although (as Wilhelm Magnus pointed out in 1974) they were already implicit in Adolf Hurwitz's work on monodromy (1891). In fact, as Magnus says, Hurwitz gave the interpretation of a braid group as the fundamental group of a configuration space (cf. braid theory), an interpretation that was lost from view until it was rediscovered by Ralph Fox and Lee Neuwirth in 1962.

Definition 1

Let D be a unit cube, so D={(x,y,z):0<x,y,z<1} on the top face of cube place n points $a_1,a_2,\ldots,a_n$ similarly place on bottom face $b_1,b_2,\ldots,b_n$, now join the points $a_1,a_2,\ldots,a_n$ with $b_1,b_2,\ldots,b_n$ by arcs $d_1,d_2,\ldots,d_n$ these arcs are disjoint and each d_i connects some a_j to b_k not connect a_j to a_k or b_j to b_k this called tangle[1].



2001 Mathematics Subject Classification, 51H10, 57H10
*Mathematics Department, Faculty of science, Tanta University, Tanta, Egypt
**Mathematics Department Faculty of science, Al-Azhar University, Egypt

Definition 2

Let M and N be two. Riemannian manifolds (not necessarily of the same dimension), a map f: M→N is said to be an **isometric folding of** M into N if, for piecewise geodesic path β: I→ M (I= [0, 1] $\subseteq$R), the induced path $f0\beta$: I→N is piecewise geodesic and of the same length as β. If f not preserve lengths then f is **topological folding**[2,3,5,6].

Definition 3

Let M and N be two Riemannian manifolds of the same dimensions, then a map **g:** M→N is said to be an **unfolding** of M into N if, for every piecewise geodesic path β: I→M (I= [0, 1] $\subseteq$R), the induced path **g**0β: I→N is piecewise geodesic but with length greater than that of β i.e. $\forall$ x,y$\in$M$\Rightarrow$d(x,y) $\leq$d(g(x),g(y))[3].

X

Main Results:

In this paper we will discuss how we can inter change the tangle to be abraid, also the local properties of the fiber will be discussed.

- A braid is special case of tangle.
- Any horizontal line must be intersecting with strings of braid at one point.

Cases of converting tangle to braid:

Case 1:

Conversion due to unfolding of fiber:

Fig (1-1)

Fig (1-2)

)

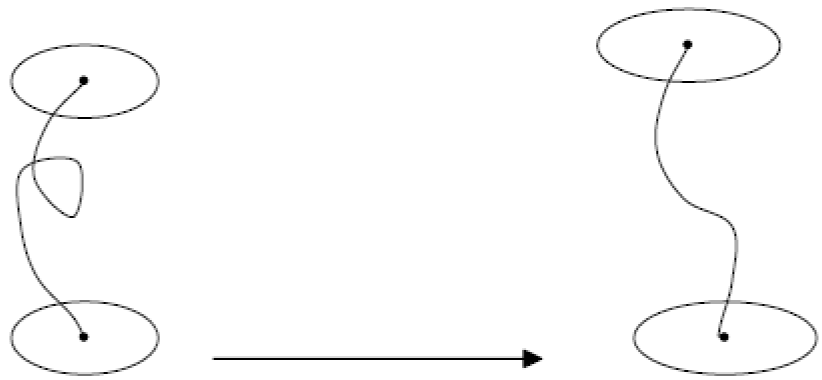


Fig.(1-3)

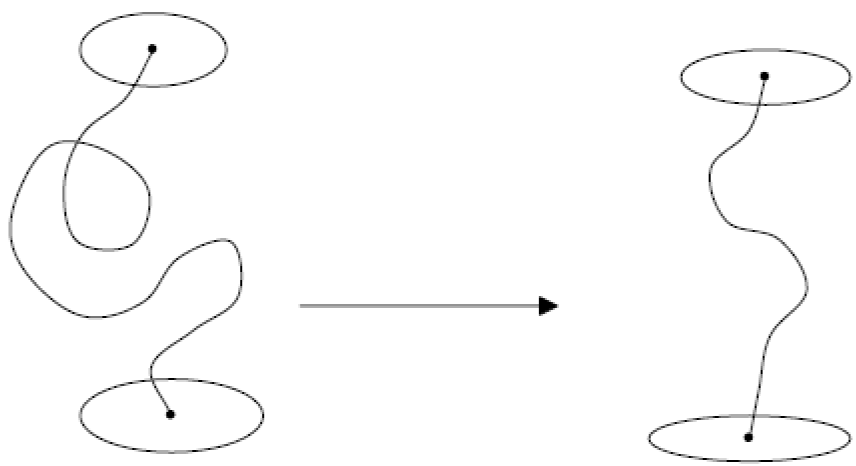


Fig (1-4)

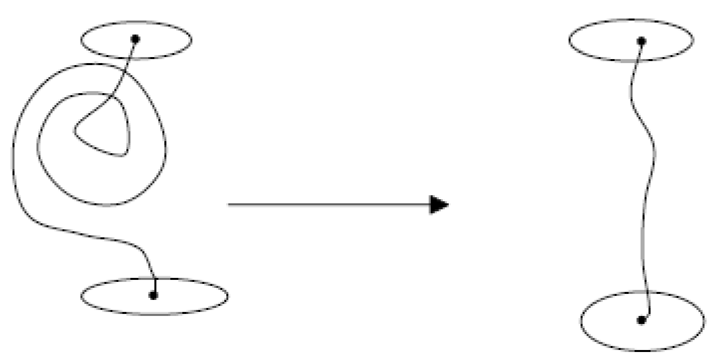


Fig. (1-5)

Case 2:
Conversion due to change position of vertices.

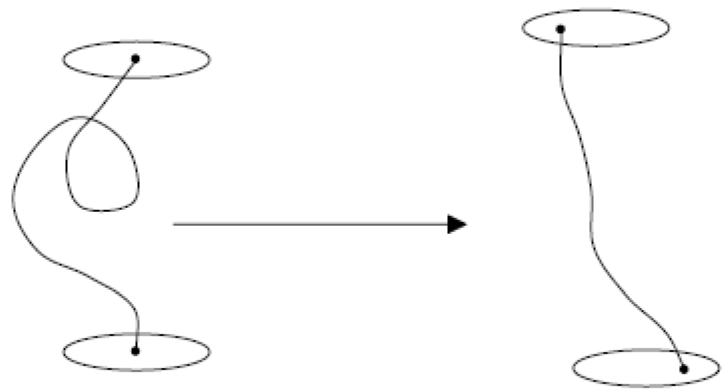


Fig. (2-1)

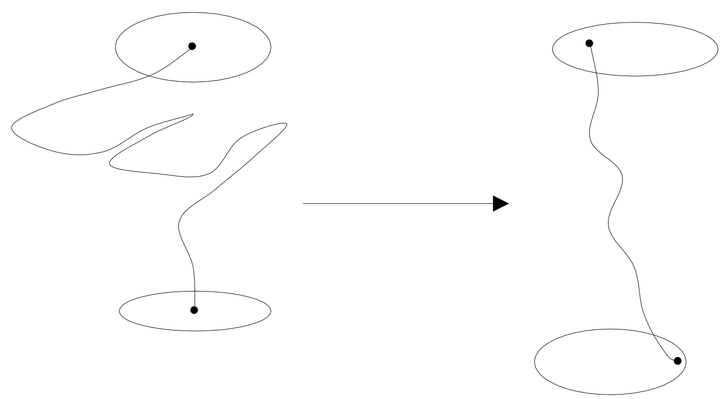


Fig. (2-2)

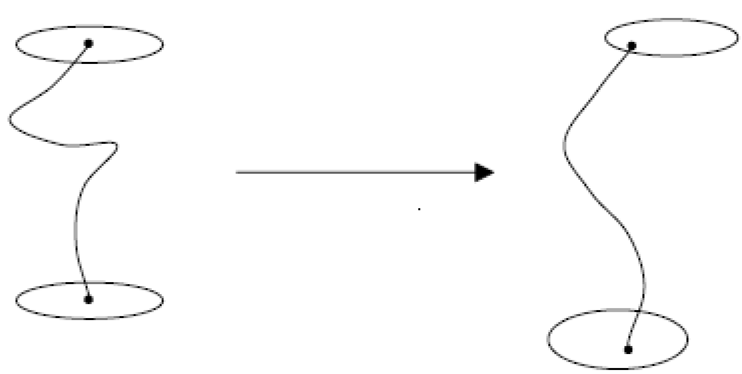
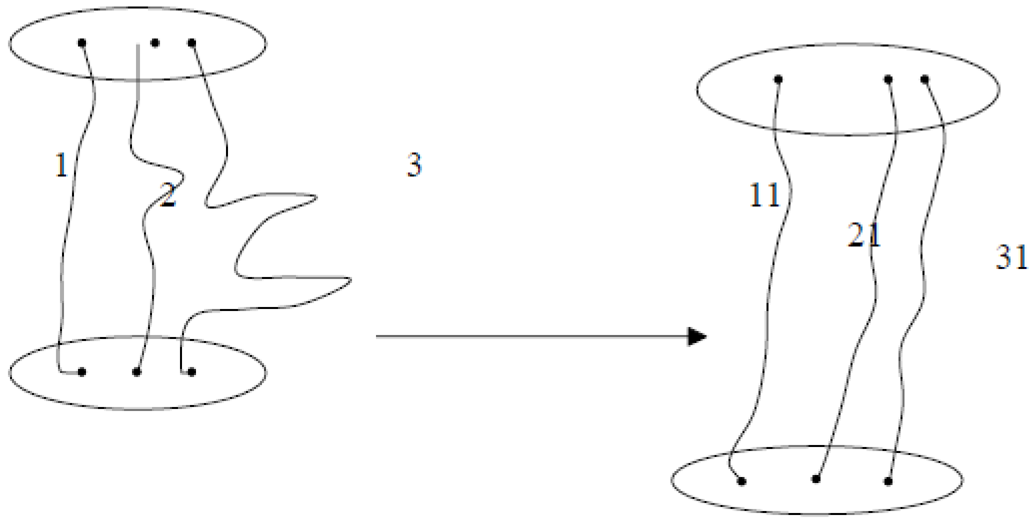


Fig. (2-3)

If we have tangle of n fibers, one of them doesn't contain tangle, we can express the other fibers by it.

Example:

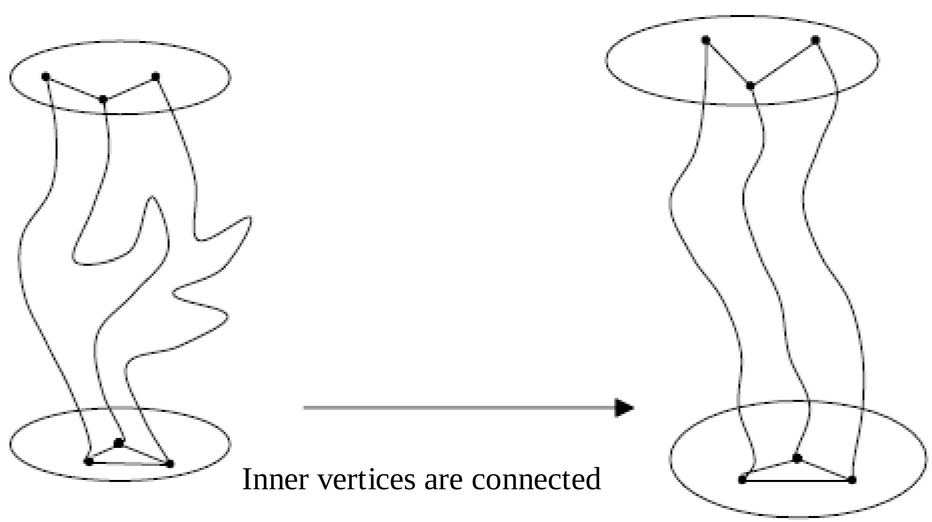


.

Inner vertices of tangle:

Inner vertices of tangles either connected or disconnected.

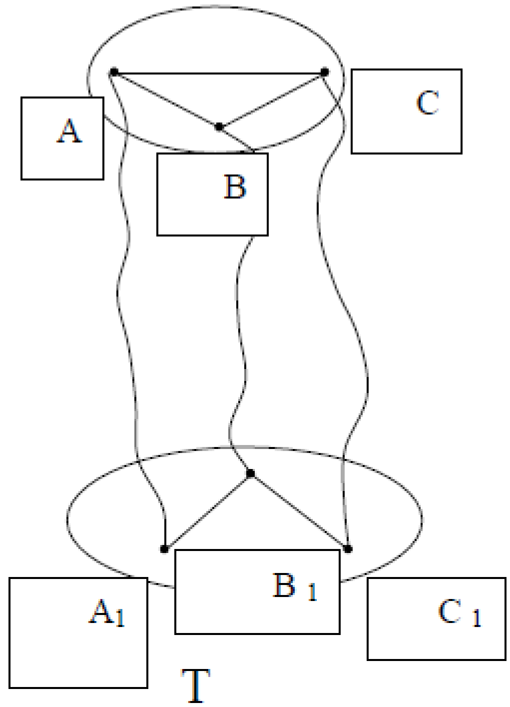
Tangle which its inner vertices are connected as previous figure, and another type as shown in the following figure.



Inner vertices are connected

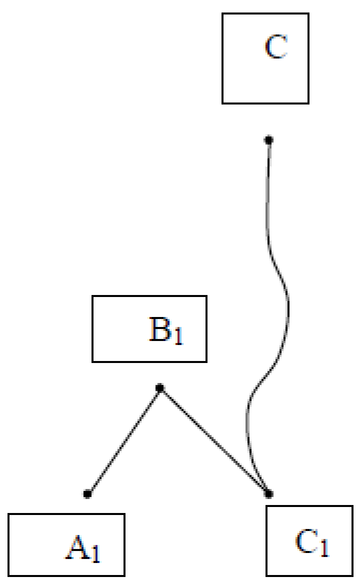
We can be introduced simple graphs from this tangle . For example, say if we want to cross from C to A_1 we have four ways these ways can be expressed by simple graph.Similarly if we want to cross from any vertex to another we have many ways these ways can be expressed it by graphs.

We now are talking about graphs from C to A_1.



First way:
The graph of this way is:

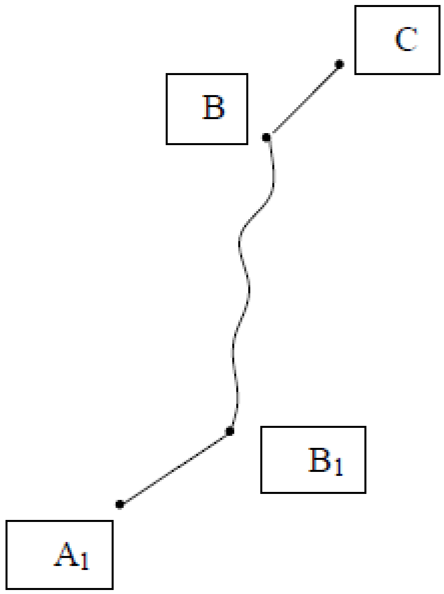
C---> C_1--->B_1--->A_1



$G_1(V_1,E_1)$

Second way is:

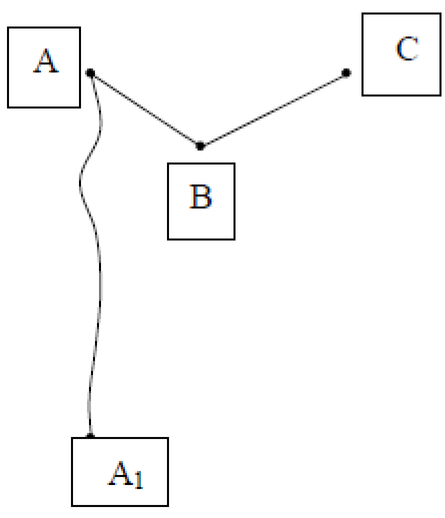
C--->B--->B_1--->A_1



$G_2(V_2,E_2)$

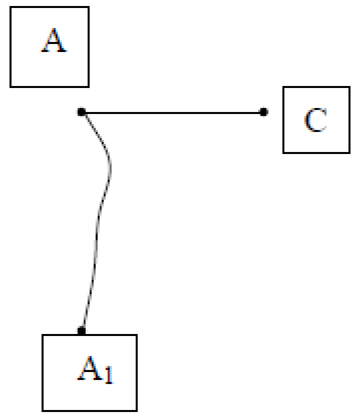
Third way is:

C--->B--->A--->A_1



$G_3(V_3,E_3)$

Furth way is:
C--->A--->A_1



$G_4(V_4,E_4)$

Adjacency and incidence matrices:

$$A\,[T] = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}, \quad I\,[T] = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

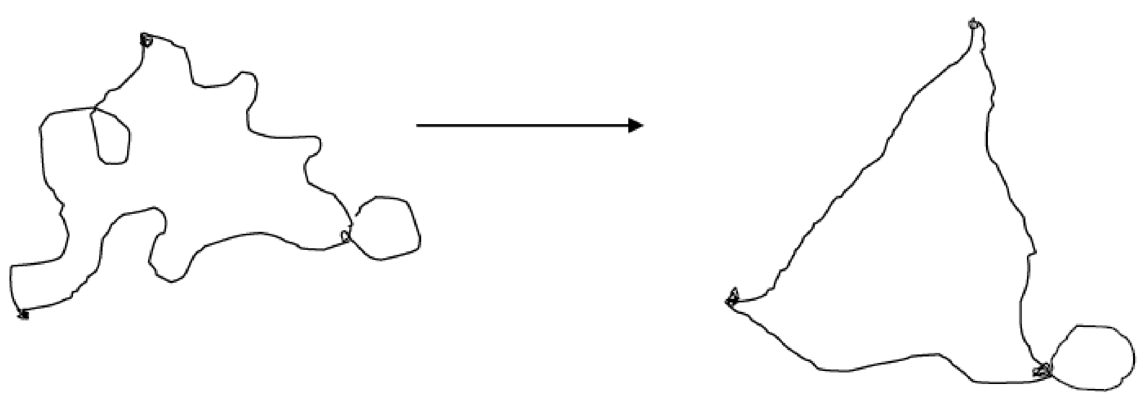
$$A\,[G_1] = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}, \quad I\,[G_1] = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$A[G_2] = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}, \; I[G_2] = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$A[G_3] = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \; I[G_3] = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A[G_4] = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad I[G_4] = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

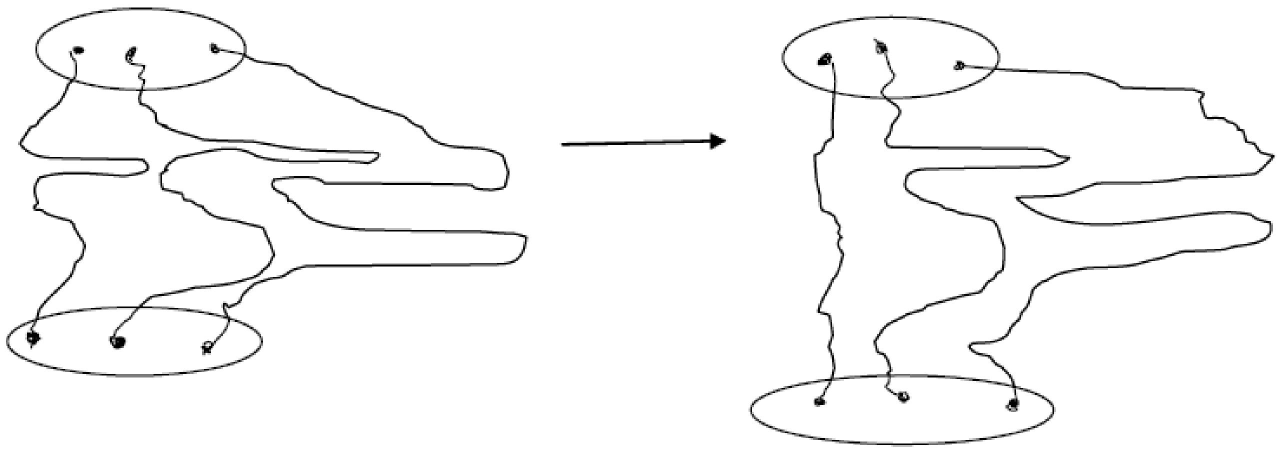
Now we explain conversion from graph whose edges are tangle into graph whose edges are braids .This conversion may occur by changing the position of vertices or unfolding fibers.



Now we study incidence and adjacency matrices.

$$A[G] = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}, I[G] = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

If we have a tangle consists of three fibers. Now we want to convert it into braid by changing on every fiber and studying the matrices on every change.



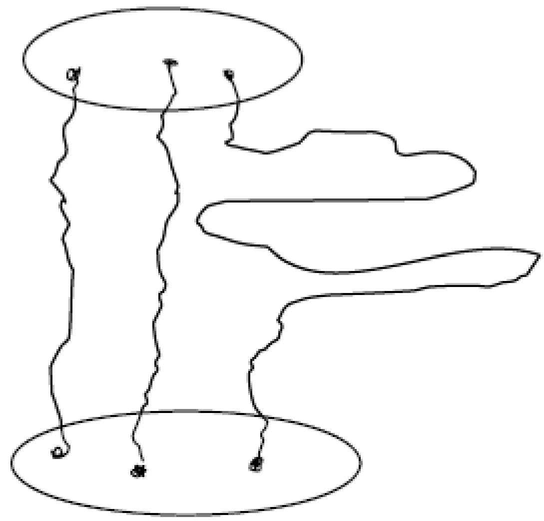
T

T_1

$$A[T] = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \quad I[T] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A[T_1] = \begin{bmatrix} 0 & 0 & 0 & 1_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \quad I[T_1] = \begin{bmatrix} 1_1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1_1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

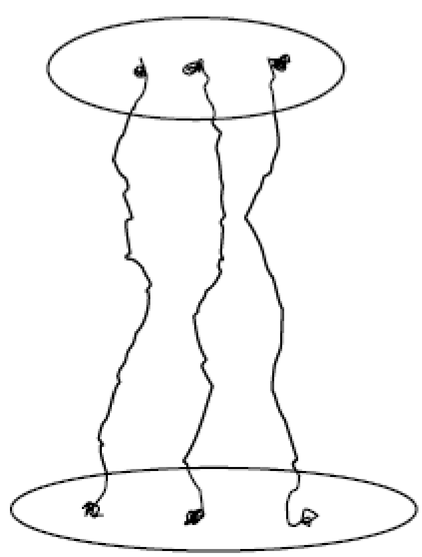
This tangle produced due to change of two fibers. Now We are studying the matrices of new tangle.



T_2

$$A\,[T_2] = \begin{bmatrix} 0 & 0 & 0 & 1_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \quad I\,[T_2] = \begin{bmatrix} 1_1 & 0 & 0 \\ 0 & 1_2 & 0 \\ 0 & 0 & 1 \\ 1_1 & 0 & 0 \\ 0 & 1_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

If we change on the third fiber, in this case we can say we convert the tangle into braid.



B

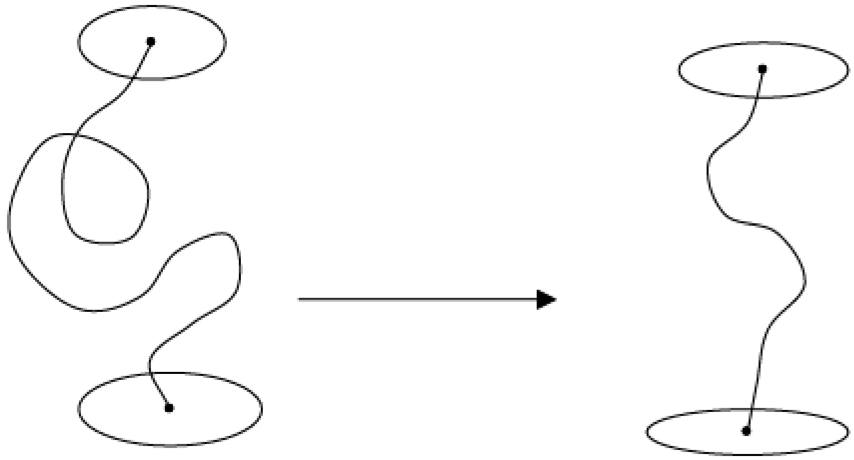
The matrices of the braid are:

$$A\,[B] = \begin{bmatrix} 0 & 0 & 0 & 1_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1_3 \\ 1_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1_3 & 0 & 0 & 0 \end{bmatrix},\quad I\,[B] = \begin{bmatrix} 1_1 & 0 & 0 \\ 0 & 1_2 & 0 \\ 0 & 0 & 1_3 \\ 1_1 & 0 & 0 \\ 0 & 1_2 & 0 \\ 0 & 0 & 1_3 \end{bmatrix}$$

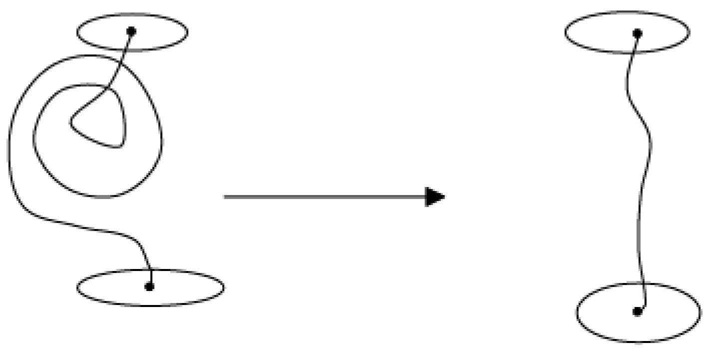
Now we represent the matrices of tangles and braids. We express the identity element of tangle which hasn't any roll by 1^0 and the tangle which has any number of rolls by 1^i where (i=1, 2, 3…).the identity element of tangle which has any number of curves by$1_r$where (r=1, 2, 3…).

$$A\,[T] = \begin{bmatrix} 0 & 1_1^0 \\ 1_1^0 & 0 \end{bmatrix},\quad I\,[T] = \begin{bmatrix} 1_1^0 \\ 1_1^0 \end{bmatrix}$$

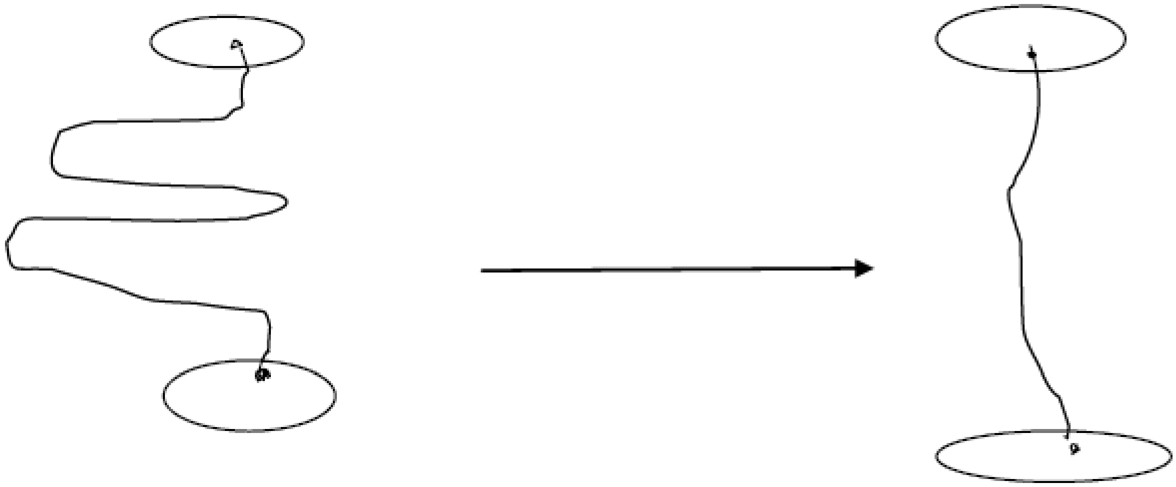
$$A\,[T] = \begin{bmatrix} 0 & 1^1 \\ 1^1 & 0 \end{bmatrix},\quad I\,[T] = \begin{bmatrix} 1^1 \\ 1^1 \end{bmatrix}$$



A [T] = $\begin{bmatrix} 0 & 1_1^2 \\ 1_1^2 & 0 \end{bmatrix}$, I [T] = $\begin{bmatrix} 1_1^2 \\ 1_1^2 \end{bmatrix}$

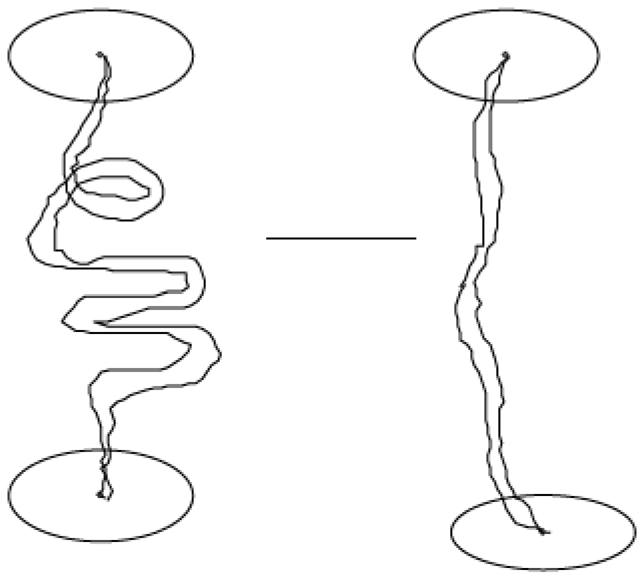


A [T] = $\begin{bmatrix} 0 & 1^3 \\ 1^3 & 0 \end{bmatrix}$, I [T] = $\begin{bmatrix} 1^3 \\ 1^3 \end{bmatrix}$

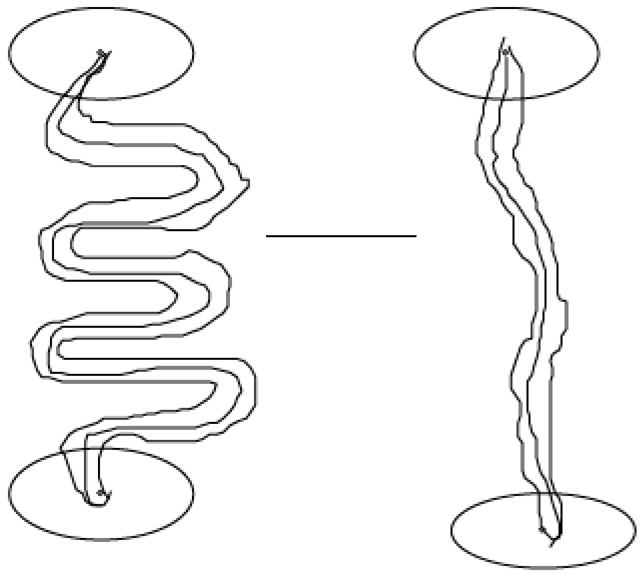


A [T] = $\begin{bmatrix} 0 & 1_3 \\ 1_3 & 0 \end{bmatrix}$, I [T] = $\begin{bmatrix} 1_3 \\ 1_3 \end{bmatrix}$

All these cases when we have one fiber. If we have n fiber then the identity element can be express by 1^s where (s=2, 3… n).



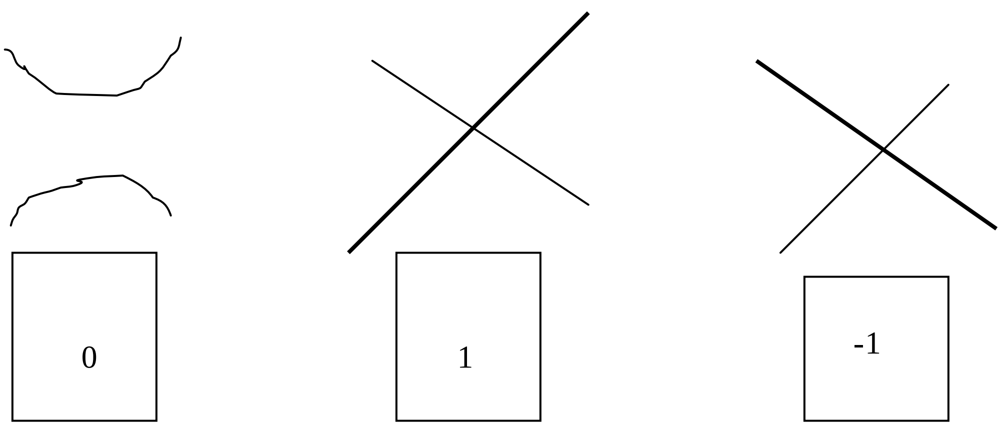
A [T] = $\begin{bmatrix} 0 & 1_4^{12} \\ 1_4^{12} & 0 \end{bmatrix}$, I [T] = $\begin{bmatrix} 1_4^{12} \\ 1_4^{12} \end{bmatrix}$



$$A\,[T] = \begin{bmatrix} 0 & 1_5^{03} \\ 1_5^{03} & 0 \end{bmatrix},\quad I\,[T] = \begin{bmatrix} 1_5^{03} \\ 1_5^{03} \end{bmatrix}$$

Elementary tangles:

Elementary tangles are:

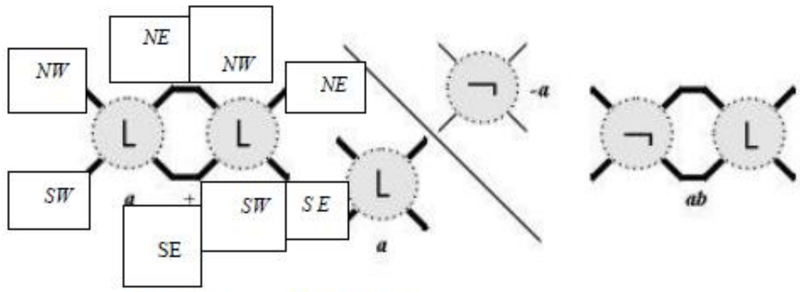


Operations on tangle:

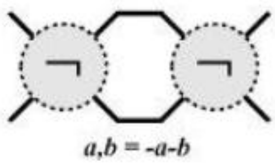
Tangles can be combined and modified by a unary operation **a---> -a** and there are three binary operations,*sum*,*product*,*ramification,* taking tangles **a, b** to new tangles **a +b, ab, and (a, b).Here –a** is the image of **a** under reflection *NW-SE* mirror, **a+b** is obtained by placing **a** and **b** side by side with **a** on the left and **b** on the right.

ab is equivalent to **(-a) +b**, and finally **a, b** is equivalent to **(-a) + (-b).** The resulting object **a**+ **b** is obtained by gluing *NE* of **a** to *NW* of **b**, and *SE* of **a** to *SW* of **b**.

Similarly product and ramification [4].



Sum and product of tangles



Ramification of tangle

Adjacency and incidence matrices:

$A[a] = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}$, $I[a] = \begin{bmatrix} I \\ I \end{bmatrix}$

$A[a^{-}] = \begin{bmatrix} 0 & I_0 \\ I_0 & 0 \end{bmatrix}$, $I[a^{-}] = \begin{bmatrix} I_0 \\ I_0 \end{bmatrix}$

$A[b] = \begin{bmatrix} 0 & I_1 \\ I_1 & 0 \end{bmatrix}$, $I[b] = \begin{bmatrix} I_1 \\ I_1 \end{bmatrix}$

$A[a+b] = \begin{bmatrix} 0 & I_2 \\ I_2 & 0 \end{bmatrix}$, $I[a+b] = \begin{bmatrix} I_2 \\ I_2 \end{bmatrix}$

$A[ab] = A[a^{-}+b] = \begin{bmatrix} 0 & I_3 \\ I_3 & 0 \end{bmatrix}$, $I[a^{-}+b] = \begin{bmatrix} I_3 \\ I_3 \end{bmatrix}$

References

M.El-Ghoul &M.M.Al-Shamiri, A study on graphs and Knots.

E. El-Kholy: Isometric and topological folding of manifold, Ph. D. Thesis, University of Southampton, UK. (1981).

M. El-Ghoul, Unfolding of Riemannian manifolds, Commun. Fac. Sci.Univ. Ankara Series, 37 (1988)

Conway, J. H. "An Enumeration of Knots and Links, and Some of Their Algebraic Properties." In J. Leech (editor), Computational Problems in Abstract Algebra. Oxford, England. Pergamum Press, pp. 329-358, 1970

P. DL-Francessco, "Folding and Coloring problem in Mathematics and physics", Bulliten of the American Mathematics Society, Vol. 37, No.3,(251-307) , July 2000

M. EL-Ghoul, " Folding of manifolds" , Ph.D. Thesis, Tanta Univ., Egypt (1985)