The Performance Comparison of LCMV and LCMN to Signals, Interference and Noise

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Abstract

To achieve the SDMA scheme of a smart antenna system, we may introduce the LCMV algorithm to set direction constraints to preserve the desired signals as well as to suppress interference simultaneously. To apply LCMV, however, we will encounter the heavy computing load as well as the unknown covariance matrix problems which make the implementation of a LCMV-based smart antenna system especially challenging. In this paper a simplified algorithm LCMN, with much less computing load, is introduced as the substitute of LCMV algorithm for smart antenna system. The performance of these two algorithms is compared. We found that LCMV is adaptive and outperform LCMN in interference suppression, while LCMN is non-adaptive and is less susceptible to noise. Moreover LCMN can achieve the same performance as that of LCMV does when the columns of constraint matrix can span signal subspace.

Keywords: Adaptive array, beamforming, LCMN, LCMV, SAS, SDMA

I. Introduction

Over the last few years the escalating demand of wireless communication services is beyond expectations. The operation of wireless communication is subject to the limited available spectrum, thus, how to increase channel capacity becomes an important topic. Spatial division multiple access (SDMA) scheme is another mean, besides that of TDMA, FDMA, CDMA and WCDMA, to expand channel capacity for better wireless communication services. To achieve the SDMA scheme, both the array systems and DSP techniques are combined to explore effectively the spatial resources, which these systems are now widely termed as smart antenna systems (SAS). The adaptive algorithms are applied to the SAS systems for digital beamforming to gain better control both the desired signals and interference, as well as, to increase channel capacity.

The linearly constrained minimum variance (LCMV) criterion for choosing array weighting vector is widely used in both narrowband and broadband beamforming[1-5]. This algorithm is aimed at minimizing the output power or variance, while subject to keep the proposed response to the desired signals. To apply LCMV, however, we will encounter not only a heavy computing load from the evaluation of covariance matrix inversion, but also this covariance matrix being processed is not known or is with time-varying characteristic such that the implementation of LCMV is especially challenging. Even one may expect the heavy computing load of matrix inversion problem may finally be solved by fast computer or DSP techniques; still, a real-time estimation of the covariance matrix is unrealistic because we will always encounter an inhomogeneous wireless communication background.

The consistent estimation of a covariance matrix is important because it will eventually degrade the performance of a LCMV-based SAS system. In this paper a simplified form of LCMV, called LCMN (linearly constrained minimum norm), is introduced as the substitute of LCMV for the SAS system and the performance of these two algorithms is compared.

The contents follow the introduction is briefly described. In section II the array model, signal parameters and the overview of LCMV and LCMN are depicted. The performance comparison of LCMV and LCMN is made at section III and IV. In section III we show that LCMV is better than LCMN on interference suppression, however the noise output of LCMN is smaller. When the columns of constrain matrix can span signal subspace, the performance of LCMV and LCMN is the same, proved in section IV. The simulation of LCMV outperform LCMN on interference suppression is demonstrated in section V. Finally we make conclusions.

II. Problem Description

A. The System Model

Assume there are K different signals, $\{ s_1(t), s_2(t), \cdots, s_K(t) \}$, which may include both the desired signals and interference and are independent among each other, impinging array from K different directions of angle of arrival (DOA) $\{\theta_1, \theta_2, \cdots, \theta_K\}$, respectively. Considered these signals are received and processed by an N-element uniform linear array with *d* spacing between array elements. We assume each element pattern is isotropic in the azimuth direction, thus the signal received at the i^{th} element is:

$$x_i(t) = \sum_{n=1}^{K} s_n(t)\exp\left(j2\pi \frac{d}{\lambda}(i-1)\cos\theta_n\right) + v_i(t) \tag{1}$$

In equation (1) θ_n is the angle measured from array end-fire direction, λ is the wavelength and $v_i(t)$ denotes the received noise. We may express the received signal in vector form as follows:

$$\mathbf{x}(t) = \sum_{n=1}^{K} \mathbf{a}(\theta_n)s_n(t) + \mathbf{v}(t) = \mathbf{D}\mathbf{s}(t)+\mathbf{v}(t). \tag{2}$$

In equation (2), $\mathbf{s}(t) = [s_1(t)\cdots s_K(t)]^T$ is the signal vector and the columns of the steering matrix **D=** $[\mathbf{a}(\theta_1) \cdots \mathbf{a}(\theta_K)]_{N\times K}$ is composed of K steering vectors associated with DOA of $\{\theta_1, \theta_2, \cdots, \theta_K\}$, respectively, and $\mathbf{v}(t)$ is the noise vector.

B. The LCMV algorithm

The LCMV algorithm is described as follows:

$$\min_{\mathbf{w}}(\mathbf{w}^H\mathbf{R}\mathbf{w}) \quad , \quad \text{subject to} \quad \mathbf{C}^H\mathbf{w} = \mathbf{g} \quad . \tag{3}$$

The weighting vector satisfied above criteria is $\mathbf{w} = \mathbf{R}^{-1}\mathbf{C}(\mathbf{C}^H\mathbf{R}^{-1}\mathbf{C})^{-1}\mathbf{g}$. In equation (3), **C** is the constraint matrix and **g** is the control vector for the proposed response to both signals and interference.

C. The LCMN Algorithm

The LCMN algorithm is described as follows:

$$\min_{\mathbf{w}}(\mathbf{w}^H\mathbf{w}) \quad , \quad \text{subject to} \quad \mathbf{C}^H\mathbf{w} = \mathbf{g} \quad . \tag{4}$$

The weighting function **w** matched above criteria is $\mathbf{w} = \mathbf{C}(\mathbf{C}^H\mathbf{C})^{-1}\mathbf{g}$. Note that both the LCMV and LCMN algorithms are designed to match the same constraint equation $\mathbf{C}^H\mathbf{w} = \mathbf{g}$, and thus the characteristics and performance difference between these two algorithms can obviously be obtained from comparing $\min_{\mathbf{w}}(\mathbf{w}^H\mathbf{R}\mathbf{w})$ vs. $\min_{\mathbf{w}}(\mathbf{w}^H\mathbf{w})$, analyzed in the next two sections.

III. Comparison of LCMV and LCMN

For simplicity, let's assume $s_1(t)$, with expected power $E(|s_1(t)|^2) = \rho^2$ and from DOA θ_1 of equation (2), is the only known and desired signal among K signals $s_1(t), \cdots, s_K(t)$. The other K-1 signals $s_2(t), \cdots, s_K(t)$ are unknown and considered to be interference. Let's define the steering vector associated with $s_1(t)$ is $\mathbf{C}_1 = \mathbf{a}(\theta_1)$. $\mathbf{D}_I = [\mathbf{a}(\theta_2) \cdots \mathbf{a}(\theta_K)]_{N\times(K-1)}$ denotes the steering matrix for interference signal vector $\mathbf{s}_I(t) = [s_2(t) \cdots s_K(t)]^T$. We can express the covariance matrix of the received signal as

$$\mathbf{R} = \rho^2 \mathbf{C}_1 \mathbf{C}_1^H + \mathbf{R}_I , \tag{5}$$

where $\mathbf{R}_I = \mathbf{D}_I \mathbf{R}_{ii} \mathbf{D}_I^H + \sigma^2 \mathbf{I}_N$ and $\mathbf{R}_{ii} = E(\mathbf{s}_I(t)\mathbf{s}_I^H(t))$. The inverse of $\mathbf{R}$ is

$$\mathbf{R}^{-1} = \mathbf{R}_I^{-1} - \frac{\rho^2 \mathbf{R}_I^{-1} \mathbf{C}_1 \mathbf{C}_1^H \mathbf{R}_I^{-1}}{1 + \rho^2 \mathbf{C}_1^H \mathbf{R}_I^{-1} \mathbf{C}_1} . \tag{6}$$

Based on the above assumption, the constraint equation is set as $\mathbf{C}_1^H \mathbf{w} = 1$. The array weighting functions based on LCMV and LCMN can be expressed, respectively, as follows:

$$\mathbf{w}_{V1} = \frac{1}{\mathbf{C}_1^H \mathbf{R}_I^{-1} \mathbf{C}_1} \mathbf{R}_I^{-1} \mathbf{C}_1 , \tag{7}$$

$$\mathbf{w}_{N1} = \frac{1}{\mathbf{C}_1^H \mathbf{C}_1} \mathbf{C}_1 . \tag{8}$$

The performance of LCMV and LCMN is compared based on their capabilities among the desired signals preservation, interference suppression and noise output level, analyzed in the following:

A. The array response to the desired signal $s_1(t)$

The output power of both LCMV and LCMN to the desired signal is the same, which is

$$E(|\mathbf{w}_{V1}^H \mathbf{C}_1 s_1(t)|^2) = \rho^2 = E(|\mathbf{w}_{N1}^H \mathbf{C}_1 s_1(t)|^2) . \tag{9}$$

B. The array response to noise

The noise output power of LCMV, N_{V1}, and LCMN, N_{N1}, is given as follows:

$$N_{V1} = E(|\mathbf{w}_{V1}^H \mathbf{v}(t)|^2) = \frac{\sigma^2}{(\mathbf{C}_1^H \mathbf{R}_I^{-1} \mathbf{C}_1)^2} \mathbf{C}_1^H \mathbf{R}_I^{-2} \mathbf{C}_1 , \tag{10}$$

$$N_{N1} = \sigma^2 \mathbf{w}_{N1}^H \mathbf{w}_{N1} = \frac{\sigma^2}{\mathbf{C}_1^H \mathbf{C}_1} . \tag{11}$$

By applying the Cauchy-Schwarz Inequality Theorem [6], we have

$$(\mathbf{C}_1^H \mathbf{R}_I^{-1} \mathbf{C}_1)^2 = (\mathbf{R}_I^{-1} \mathbf{C}_1)^H \mathbf{C}_1 \mathbf{C}_1^H (\mathbf{R}_I^{-1} \mathbf{C}_1) \le (\mathbf{R}_I^{-1} \mathbf{C}_1)^H (\mathbf{R}_I^{-1} \mathbf{C}_1) \mathbf{C}_1^H \mathbf{C}_1 = (\mathbf{C}_1^H \mathbf{R}_I^{-2} \mathbf{C}_1)^H (\mathbf{C}_1^H \mathbf{C}_1) . \tag{12}$$

This implies $N_{N1} \le N_{V1}$.

C. The array response to interference

1). The output power of LCMV to interference plus noise is

$$(J + N)_{V1} = \mathbf{w}_{V1}^H \mathbf{R}_I \mathbf{w}_{V1} = \frac{1}{\mathbf{C}_1^H \mathbf{R}_I^{-1} \mathbf{C}_1} . \tag{13}$$

2). The output power of LCMN to interference plus noise is

$$(J + N)_{N1} = \mathbf{w}_{N1}^H \mathbf{R}_I \mathbf{w}_{N1} = \frac{1}{(\mathbf{C}_1^H \mathbf{C}_1)^2} \mathbf{C}_1^H \mathbf{R}_I \mathbf{C}_1 . \tag{14}$$

From the result of equation (15) given below, we have $(\mathrm{J+N})_{\mathrm{V1}} \le (\mathrm{J+N})_{\mathrm{N1}}$ and we can conclude that $\mathrm{J}_{\mathrm{V1}} \le \mathrm{J}_{\mathrm{N1}}$ is true due to the fact that both $\mathrm{N}_{\mathrm{N1}} \le \mathrm{N}_{\mathrm{V1}}$ and $(\mathrm{J+N})_{\mathrm{V1}} \le (\mathrm{J+N})_{\mathrm{N1}}$ are verified.

$$
\begin{aligned}
(\mathbf{C}_1^{\mathrm{H}}\mathbf{C}_1)^2 &= \mathbf{C}_1^{\mathrm{H}}\mathbf{R}_{\mathrm{I}}^{-1/2}\mathbf{R}_{\mathrm{I}}^{1/2}\mathbf{C}_1\mathbf{C}_1^{\mathrm{H}}\mathbf{R}_{\mathrm{I}}^{1/2}\mathbf{R}_{\mathrm{I}}^{-1/2}\mathbf{C}_1 \\
&= (\mathbf{R}_{\mathrm{I}}^{-1/2}\mathbf{C}_1)^{\mathrm{H}}(\mathbf{R}_{\mathrm{I}}^{1/2}\mathbf{C}_1)(\mathbf{R}_{\mathrm{I}}^{1/2}\mathbf{C}_1)^{\mathrm{H}}(\mathbf{R}_{\mathrm{I}}^{-1/2}\mathbf{C}_1) \le (\mathbf{C}_1^{\mathrm{H}}\mathbf{R}_{\mathrm{I}}^{-1/2}\mathbf{R}_{\mathrm{I}}^{-1/2}\mathbf{C}_1)(\mathbf{C}_1^{\mathrm{H}}\mathbf{R}_{\mathrm{I}}^{1/2}\mathbf{R}_{\mathrm{I}}^{1/2}\mathbf{C}_1) \\
&= (\mathbf{C}_1^{\mathrm{H}}\mathbf{R}_{\mathrm{I}}^{-1}\mathbf{C}_1)(\mathbf{C}_1^{\mathrm{H}}\mathbf{R}_{\mathrm{I}}\mathbf{C}_1).
\end{aligned} \tag{15}
$$

IV. Comparison of LCMV and LCMN under span(C)=span(D) Case

When the covariance matrix **R** is known, $\mathbf{R}=\mathbf{D}\mathbf{R}_{\mathrm{S}}\mathbf{D}^{\mathrm{H}}+\sigma^2\mathbf{I}_{\mathrm{N}}$, where $\mathbf{R}_{\mathrm{S}} \equiv \mathrm{E}[\mathbf{s}(t)\mathbf{s}^{\mathrm{H}}(t)]$ and σ^2 the noise power. The covariance matrix **R** can be eigen-decomposed to be $\mathbf{R}=\mathbf{Q}\Lambda\mathbf{Q}^{\mathrm{H}}+\sigma^2\mathbf{I}_{\mathrm{N}}$, where $\mathbf{Q}=[\mathbf{q}_1\cdots\mathbf{q}_{\mathrm{K}}]$ is composed of the K eigenvectors associated with the largest K eigenvalues $\lambda_1,\cdots,\lambda_{\mathrm{K}}$ of **R**, respectively. The diagonal matrix Λ is

$$
\Lambda = \begin{bmatrix} \lambda_1-\sigma^2 & & 0 \\ & \ddots & \\ 0 & & \lambda_{\mathrm{K}}-\sigma^2 \end{bmatrix}. \tag{16}
$$

The inverse of **R** can be expressed as

$$
\mathbf{R}^{-1} = \frac{\mathbf{I}_{\mathrm{N}}}{\sigma^2} - \frac{1}{\sigma^2}\mathbf{Q}\begin{bmatrix} (\lambda_1-\sigma^2)/\lambda_1 & & 0 \\ & \ddots & \\ 0 & & (\lambda_{\mathrm{K}}-\sigma^2)/\lambda_{\mathrm{K}} \end{bmatrix}\mathbf{Q}^{\mathrm{H}}. \tag{17}
$$

Note that the column vectors of **Q** form a basis for the signal subspace which thus can span all column vectors of **D**. This implies that each column vector of **D** can be expressed as the combination of column vectors of **Q**, expressed as follows:

$$
\mathbf{a}(\theta_{\mathrm{i}}) = \mathbf{q}_1\alpha_{\mathrm{i1}} + \cdots + \mathbf{q}_{\mathrm{K}}\alpha_{\mathrm{iK}} = \mathbf{Q}\mathbf{A}_{\mathrm{i}}, \quad \mathrm{i}=1,\ldots,\mathrm{K}, \tag{18}
$$

where $\mathbf{A}_{\mathrm{i}}=[\alpha_{\mathrm{i1}}\cdots\alpha_{\mathrm{iK}}]^{\mathrm{T}}$ is the coordinate vector of $\mathbf{a}(\theta_{\mathrm{i}})$ with respect to **Q**. Consequently we can define coordinate matrix $\mathbf{A}=[\mathbf{A}_1\cdots\mathbf{A}_{\mathrm{K}}]_{\mathrm{K}\times\mathrm{K}}$ such that $\mathbf{D}=[\mathbf{a}(\theta_1)\cdots\mathbf{a}(\theta_{\mathrm{K}})]=\mathbf{Q}\mathbf{A}$.

When **R** is known, we may choose the steering matrix **D** to serve as the constraint matrix **C**, which means all the K incoming signals can be governed by means of setting constraints. Obviously, this is under the *span*(**C**) = *span*(**D**) condition, which *span*(**C**) denotes the spanning set of matrix. We would add subscript to both weighting functions to draw distinction between LCMV and LCMN, which $\mathbf{w}_{\mathrm{V}}=\mathbf{R}^{-1}\mathbf{D}(\mathbf{D}^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{D})^{-1}\mathbf{g}$ denotes the LCMV-based weighting function and that for the LCMN algorithm is $\mathbf{w}_{\mathrm{N}}=\mathbf{D}(\mathbf{D}^{\mathrm{H}}\mathbf{D})^{-1}\mathbf{g}$. The array outputs based on LCMV and LCMN weighting functions are compared in the following:

A. The output power of LCMV and LCMN

1). The LCMV-based array output power is

$$
\mathrm{E}(|\mathbf{w}_{\mathrm{V}}^{\mathrm{H}}\mathbf{x}(t)|^2) = \mathbf{w}_{\mathrm{V}}^{\mathrm{H}}\mathbf{R}\mathbf{w}_{\mathrm{V}} = \mathbf{g}^{\mathrm{H}}(\mathbf{D}^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{D})^{-1}\mathbf{g} = \mathbf{g}^{\mathrm{H}}\mathbf{A}^{-1}\begin{bmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_{\mathrm{K}} \end{bmatrix}\mathbf{A}^{-\mathrm{H}}\mathbf{g}, \tag{19}
$$

where

$$
\mathbf{D}^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{D} = \mathbf{A}^{\mathrm{H}}\mathbf{Q}^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{Q}\mathbf{A} = \mathbf{A}^{\mathrm{H}}\begin{bmatrix} 1/\lambda_1 & & 0 \\ & \ddots & \\ 0 & & 1/\lambda_{\mathrm{K}} \end{bmatrix}\mathbf{A},
$$

and

$$
(\mathbf{D}^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{D})^{-1} = \mathbf{A}^{-1}\begin{bmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_{\mathrm{K}} \end{bmatrix}\mathbf{A}^{-\mathrm{H}}.
$$

2). The LCMN-based array output power is

$$\mathbf{w}_N^H\mathbf{R}\mathbf{w}_N = \mathbf{g}^H(\mathbf{D}^H\mathbf{D})^{-1}(\mathbf{D}^H\mathbf{R}\mathbf{D})(\mathbf{D}^H\mathbf{D})^{-1}\mathbf{g} = \mathbf{g}^H\mathbf{A}^{-1}\begin{bmatrix}\lambda_1 & & 0\\ & \ddots & \\ 0 & & \lambda_K\end{bmatrix}\mathbf{A}^{-H}\mathbf{g} = \mathbf{w}_V^H\mathbf{R}\mathbf{w}_V\,. \tag{20}$$

Equation (20) is derived by applying equations (21) and (22) in the following:

$$(\mathbf{D}^H\mathbf{D})^{-1} = (\mathbf{A}^H\mathbf{A})^{-1}\,. \tag{21}$$

$$\mathbf{D}^H\mathbf{R}\mathbf{D} = \mathbf{A}^H\mathbf{Q}^H(\mathbf{Q}\Lambda\mathbf{Q}^H + \sigma^2\mathbf{I}_N)\mathbf{Q}\mathbf{A} = \mathbf{A}^H\begin{bmatrix}\lambda_1 & & 0\\ & \ddots & \\ 0 & & \lambda_K\end{bmatrix}\mathbf{A}\,. \tag{22}$$

Equation (20) implies that the array output power based on LCMV is the same as that of LCMN-based.

B. The response of LCMN and LCMV to the desired signal plus interference

The outputs of LCMN and LCMV to the desired signal and interference are the same, given in equation (23), because both algorithms are set to meet the same constraint equation $\mathbf{D}^H\mathbf{w} = \mathbf{g}$.

$$\mathrm{E}(\left|\mathbf{w}_V^H\mathbf{D}\mathbf{s}(t)\right|^2) = \mathbf{g}^H\mathbf{R}_S\mathbf{g} = \mathrm{E}(\left|\mathbf{w}_N^H\mathbf{D}\mathbf{s}(t)\right|^2)\,. \tag{23}$$

C. The response of LCMN and LCMV to noise

1). The LCMV-based noise output power is

$$\mathrm{N}_V = \mathrm{E}(\left|\mathbf{w}_V^H\mathbf{v}(t)\right|^2) = \sigma^2\mathbf{g}^H(\mathbf{D}^H\mathbf{R}^{-1}\mathbf{D})^{-1}(\mathbf{D}^H\mathbf{R}^{-2}\mathbf{D})(\mathbf{D}^H\mathbf{R}^{-1}\mathbf{D})^{-1}\mathbf{g} = \sigma^2\mathbf{g}^H(\mathbf{A}^H\mathbf{A})^{-1}\mathbf{g} = \sigma^2\mathbf{g}^H(\mathbf{D}^H\mathbf{D})^{-1}\mathbf{g}\,. \tag{24}$$

To derive equation (24), we use

$$\mathbf{R}^{-2} = \frac{\mathbf{I}_N}{\sigma^4} + \frac{1}{\sigma^4}\mathbf{Q}\begin{bmatrix}(\sigma^4-\lambda_1^2)/\lambda_1^2 & & 0\\ & \ddots & \\ 0 & & (\sigma^4-\lambda_K^2)/\lambda_K^2\end{bmatrix}\mathbf{Q}^H,$$

and

$$\mathbf{D}^H\mathbf{R}^{-2}\mathbf{D} = \mathbf{A}^H\mathbf{Q}^H\mathbf{R}^{-2}\mathbf{Q}\mathbf{A} = \mathbf{A}^H\begin{bmatrix}1/\lambda_1^2 & & 0\\ & \ddots & \\ 0 & & 1/\lambda_K^2\end{bmatrix}\mathbf{A}\,.$$

2). The LCMN-based noise output power is

$$\mathrm{N}_N = \mathrm{E}(\left|\mathbf{w}_N^H\mathbf{v}(t)\right|^2) = \sigma^2\mathbf{g}^H(\mathbf{D}^H\mathbf{D})^{-1}\mathbf{g}\,. \tag{25}$$

From equations (24) and (25), we know that the array noise output power based on LCMV is the same as that of the LCMN-based.

V. Simulation Results

We consider a uniformly linear array with 14 sensors which the spacing between array elements is half-wavelength. Five signals impinging array from the DOA of $\theta = 40^0, 70^0, 80^0, 100^0$ and 130^0 are assumed, which they are independent among each other. The constraint equation is designed to set the former three signals from $\theta = 40^0, 70^0$ and 80^0 under the $\mathbf{g} = [1\ 0\ 0]^T$ constraint. Two simulation results are presented to compare the interference suppression performance of LCMV and LCMN.

In Fig.1 example we plot the array output of signal from $\theta = 100^0$ which is not set under any constraint, and this figure shows that LCMV can suppress this interference; while the LCMN-based scheme cannot achieve the desired performance. Fig.2 demonstrates the array output of signal from $\theta = 80^0$ which is under null-constraint. It is clear that both LCMN and LCMV can suppress this interference with similar performance.

VI. Discussion and Conclusion

To know the characteristics of LCMV, we may decompose the LCMV-based weighting function into the adaptive and non-adaptive two parts from the generalized sidelobe canceller (GSC) architecture point of view, which LCMN is actually the non-adaptive part of LCMV [5]. The adaptive part of LCMV provides the flexibility of this algorithm to deal with the variation of signals or outer environment. One may see this fact by checking the existence of signal covariance matrix **R** in LCMV-based weighting function, which is $\mathbf{w}_V = \mathbf{R}^{-1}\mathbf{D}(\mathbf{D}^H\mathbf{R}^{-1}\mathbf{D})^{-1}\mathbf{g}$; while **R** does not appear to the LCMN-based weighting function $\mathbf{w}_N = \mathbf{D}(\mathbf{D}^H\mathbf{D})^{-1}\mathbf{g}$. This is the reason why LCMN is non-adaptive and cannot response to both the variation of signals and environment. Thus, the LCMV algorithm has better performance in interference suppression than the LCMN-based system under the general *span*(**C**) $\subset$ *span*(**D**) condition, even though the unknown or additional interference is not proposed to be under the null-constraint, demonstrated in the Fig.2. However, when all the incoming signal information can be obtained and is set under the constraint equations, the performance of LCMN will achieve certainly to the same as that of LCMV does because the left unknown part is noise. This is the case of *span*(**C**)=*span*(**D**) condition discussed in section IV.

We conclude that the LCMV–based is more complex and outperform the LCMN-based system by involving and processing the covariance matrix information for array weighting function evaluation. We may prefer LCMV when both the covariance matrix information can be obtained and the computing load is not a great concern. To the contrary, LCMN is worthy of the simplicity for implementation and less computing load.

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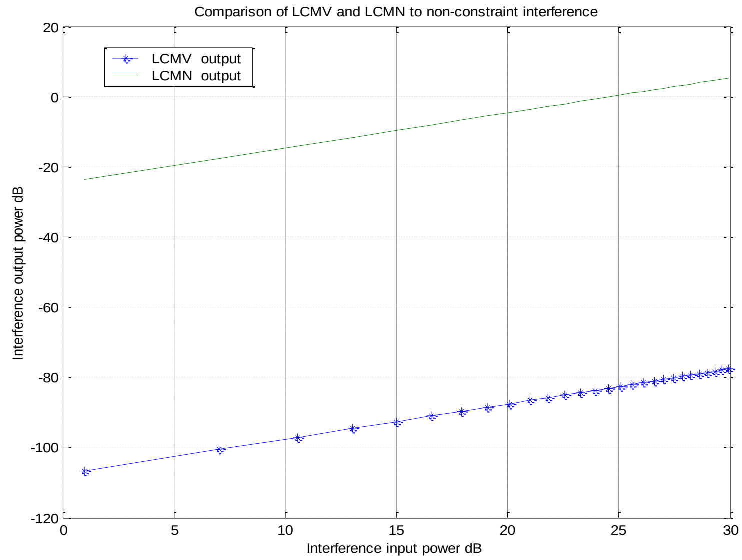

Fig. 1: The comparison of LCMV and LCMN to the non-constraint interference at DOA Of $\theta = 100^0$, which shows that LCMV can suppress it while LCMN can not.

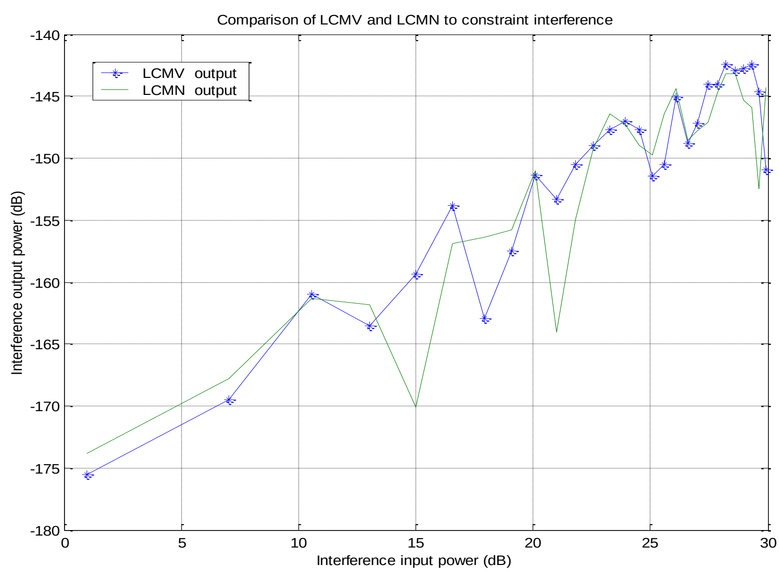

Fig.2: The comparison of LCMV and LCMN to the null-constraint interference at DOA $\theta = 80^0$ which shows that both LCMV and LCMN can suppress it with similar performance