Parameter Estimation by ANFIS in Cases Where Outputs are Non-Symmetric Fuzzy Numbers

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Abstract

Regression analysis is an area of statistics that deals with the investigation of the dependence of a variable upon one or more variables. Recently, much research has studied fuzzy estimation. There are some approaches existing in the literature for the estimation of the fuzzy regression model. Two of them are frequently used in parameter estimation, one of which is proposed by Tanaka et al [21] and known as linear programming approach and the other is fuzzy least square approach [17]. The fuzzy inference system forms a useful computing framework based on the concepts of fuzzy set theory, fuzzy reasoning, and fuzzy if-then rules. The fuzzy inference system is a powerful function approximator. There are several different types of fuzzy inference systems developed for function approximation. The Adaptive-Network Based Fuzzy Inference System (ANFIS) is a neural network architecture that can solve any function approximation problem. In this study we will use the ANFIS for parameter estimation and propose an algorithm, in cases where outputs are non-symmetric fuzzy numbers. In this algorithm the error measure is defined as the difference between the estimated outputs which are obtained by adaptive networks and the target outputs. In order to obtain the difference between two fuzzy numbers, some fuzzy ranking methods must be used to define the operator {-}. There are many fuzzy ranking methods for the measuring of the difference between the two fuzzy numbers in literature. In this work, the method of Chang and Lee [2], which is based on the concept of overall existence, will be used.

Keywords: ANFIS, OM index, Fuzzy Regression

1. Introduction

The fuzzy regression method can be used to obtain unknown parameters of regression models based on fuzzy data. This method has been introduced by Tanaka et al [21]. Several works have been published by different authors. In a study of Diamond [8] several models for simple least-squares fitting of fuzzy-valued data are developed and criteria are given for when fuzzy data sets can be fitted to the models. Ishibuchi and Nii [12] explained several versions of fuzzy regression methods based on linear fuzzy models with asymmetrical triangular fuzzy coefficients. In the paper of Kao and Chyu [17] the concept of least squares which is widely applied in the classical regression analysis is adopted to determine the regression coefficients. Hong and Hwang [11] extended Diamond's models to multi variable cases and derived efficient solutions for fuzzy multivariable regression models. D'Urso [9] suggested fuzzy regression models with crisp or fuzzy inputs and crisp or fuzzy output. A method for hybrid fuzzy least-squares regression is developed. Xu and Li [23] developed a fuzzy analogue by a distance defined on a fuzzy number space, and proposed a fuzzy multivariable linear regression model. In a study of Chang and Lee [3] a generalized fuzzy weighted least-squares regression is proposed.

In classical regression analysis, it is assumed that the observations come from a single class in a data cluster and the simple functional relationship between the dependent and independent variables can be expressed using the general model; $Y = f(X) + \varepsilon$. However; a data cluster may consist of a combination of observations that have different distributions derived from different clusters. When faced with issues of estimating a regression model for inputs that have been derived from different distributions, this regression model is termed as the 'switching regression model' and it is expressed with $Y^L = f^L(X) + \varepsilon^L$ $(L = \prod_{i=1}^{p} l_i)$. Here l_i indicates the class number of each independent variable and *p* is the indicative of the number of independent variables [10,16,18,19]. To constitute this model, the first step is to determine class numbers related to independent variables. There are methods that suggest the class numbers of independent variables heuristically. In this study, in defining the optimal class number, the use of suggested validity criteria for fuzzy clustering has been applied. Different studies examining fuzzy clustering and validity criteria exist in literature. We will use the Xie-Beni index to determine the optimal class numbers. Optimal value of class numbers l_i, $(l_i = 2,\ l_i = 3, \ldots, l_i = \max)$ can be obtained by minimizing the fuzzy clustering validity function S_i. This function is proposed by Xie and Beni [22].

Neural network enabling the use of fuzzy inference system for fuzzy regression analysis is known as adaptive network. There are many studies on the usage of the adaptive network to parameter the prediction for fuzzy regression model. Chang and Lee studied on fuzzy adaptive network approach for fuzzy regression analysis [6] and they studied on both fuzzy adaptive networks and switching regression model [5]. Jang, J. R. studied on the adaptive networks based on fuzzy inference system [16]. In 1985, in the study of Takagi and Sugeno, the method of identification of a system using its input-output data was shown [20]. James and Donalt, studied on fuzzy regression by neural network [15]. Erbay and Apaydın studied on parameter estimation in cases where independent variables come from exponential distribution by using fuzzy adaptive network [10].

The fuzzy inference system forms a useful computing framework based on the concepts of fuzzy set theory, fuzzy reasoning, and fuzzy if-then rules. ANFIS is a neural network architecture that can solve any function approximation problem. In this study we will use the ANFIS for parameter estimation and propose an algorithm in case where outputs are non-symmetric fuzzy number. Since the outputs are non-symmetric fuzzy number in this study we will used the fuzzy ranking method. Comparison or ranking of fuzzy numbers is very important for practical applications. To measure the difference between two fuzzy numbers there are various fuzzy ranking methods based on different approaches or different points of view which have been proposed in literature. Several reviews have also appeared in [1]. In this work, the method of Chang and Lee [2], which is based on the concept of overall existence, will be used.

The remainder of the paper is organized as follows. In Section 2, the difference between the non-symmetric fuzzy numbers is obtained by using an OM index for error measure. Section 3 introduces fuzzy linear regression analysis. The general information about fuzzy inference system and ANFIS are given in Section 4. In Section 5, which is the main part of this article, special ANFIS and an algorithm for parameter estimation by ANFIS in case where outputs are non-symmetric fuzzy number is given. A numerical example is given in Section 6. And finally a discussion and conclusion are provided in Section 7.

2. Difference between the Non-Symmetric Fuzzy Numbers for Error Measure

The error measure is defined as the difference between the estimated outputs which are obtained by adaptive network and the target outputs. In order to obtain the difference between the two fuzzy numbers, some fuzzy ranking method must be used to define the operator {-}. This is because fuzzy numbers are sets, not crisp numbers. There are many fuzzy ranking methods for measuring the difference between the two fuzzy numbers in literature. In this work, the method of Chang and Lee, which is based on the concept of overall existence, will be used. In order to use all the information available, all the existence levels *w* must be considered. When the *A* and *B* are two different fuzzy sets, this overall existence index can be defined as follows;

$$I = \int_0^1 g(\{\mu_A^{-1}(w)\})dw - \int_0^1 g(\{\mu_B^{-1}(w)\})dw \tag{1}$$

Here,

μ_A, μ_B : Membership functions of fuzzy sets *A* and *B*,

$\mu_A^{-1}(w)$, $\mu_B^{-1}(w)$: Ordinary subsets, denoting the inverse images of the membership functions with $w \in (0,1]$, i.e.,

$$\{\mu_A^{-1}(w)\} = \{x : \mu_A(x) = w\} \; ,$$
$$\{\mu_B^{-1}(w)\} = \{y : \mu_B(y) = w\} \; , \quad x \, , \, y \in \Re,$$

$g(\{.\})$ is a function of the inverse of membership functions. In this state the difference between *A* and *B* fuzzy sets is defined as,

$$d(A,B) = \int_0^1 g_A(\{\mu_A^{-1}(w)\})dw - \int_0^1 g_B(\{\mu_B^{-1}(w)\})dw \tag{2}$$

The special measure of each fuzzy set is defined as;

$$OM(A_i) = \int_0^1 g_i(\{\mu_A^{-1}(w)\})dw \tag{3}$$

Where, $(\{\mu_A^{-1}(w)\}) = W(w)[\chi_1(w)x_i^{'}(w) + \chi_2(w)x_i^{''}(w)]$

When the left side of non-symmetric triangular fuzzy number *A* denoted as A_l and the right side is denoted as A_r; $x^{'}(w)$ and $x^{''}(w)$ are the inverse images of the left reference and the right reference respectively, then

$$x^{'}(w) = \mu_{A_l}^{-1}(w) \quad and \quad x^{''}(w) = \mu_{A_r}^{-1}(w)$$

By these definitions, Eq. (3) now becomes

$$OM(A) = \int_0^1 W(w)[\chi_1(w)\mu_{A_l}^{-1}(w) + \chi_2(w)\mu_{A_r}^{-1}(w)]dw \tag{4}$$

Where, $\chi_1(w)$ *and* $\chi_2(w)$, are the weighting measures and must be determined subjectively by the decision maker, and these measures ensure the following conditions,

$$\chi_1(w) + \chi_2(w) = 1 \text{ and } \chi_1(w) \, , \, \chi_2(w) \in (0,1] \, .$$

and weight $W(w)$ is expressed by [2],

$$W(w) = \frac{w}{\frac{1}{2}(w_{hgt}^{*})^2} \tag{5}$$

For simplicity, when the weighting measures are $\chi_1(w) = \chi_2(w) = 1/2$ *and* $W(w) = 1$, the membership function of the triangular fuzzy number *A* is the definition by,

$$\mu_A(x) = \begin{cases} 0 & , \; x < (a-\alpha) \\ \dfrac{x-(a-\alpha)}{a-(a-\alpha)} & , \; (a-\alpha) \le x < a \\ \dfrac{(a+\beta)-x}{(a+\beta)+a} & , \; a \le x < a+\beta \\ 0 & , \; x > a+\beta \end{cases} \tag{6}$$

For the determination of an overall existence measurement of a non-symmetric triangular fuzzy number $A=(a,\alpha,\beta)$, which is given in Figure 1, the inverse images of the membership function is determinate by using the Eq. (6) as follows,

$$\begin{aligned} \frac{x-(a-\alpha)}{\alpha} &= w \Rightarrow x' = w\alpha + a - \alpha \\ \frac{(a+\beta)-x}{\beta} &= w \Rightarrow x'' = a + \beta - w\beta \end{aligned} \tag{7}$$

And the values of the inverse images which are obtained from the Eq. (7) are placed into the Eq. (4) then the index *OM* of non-symmetric triangular fuzzy number *A* is

$$OM(A)=\int_0^1 1\left[\frac{1}{2}\left(w\alpha+a-\alpha\right)+\frac{1}{2}\left(a+\beta-w\beta\right)\right]dw \tag{8}$$

And from the solution of this integral the index *OM* of *A* is

$$OM(A)=\frac{(4a-\alpha+\beta)}{4} \tag{9}$$

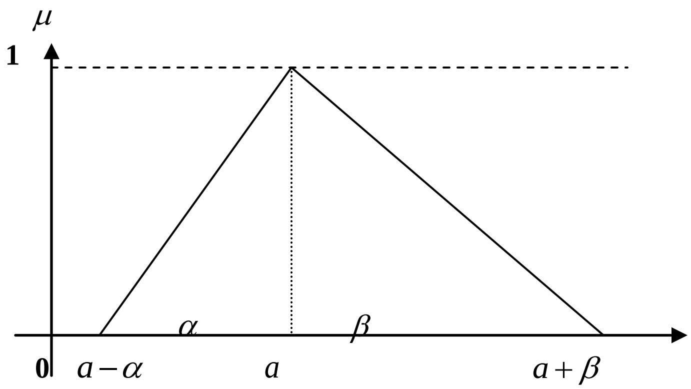

Figure 1: $A=(a,\alpha,\beta)$ **Non-Symmetric Triangular Fuzzy Number**

When the A and B are non-symmetric fuzzy numbers, which are defined by $A=(a,\alpha,\beta)$ and $B=(b,\eta,\delta)$, from the Eq. (2),(3) and (9) the difference between *A* and *B* is defined by

$$\begin{aligned} d(A,B) &= \int_0^1 g_A(\{\mu_A^{-1}(w)\})dw - \int_0^1 g_B(\{\mu_B^{-1}(w)\})dw \\ &= OM(A)-OM(B) \\ &= \frac{(4a-\alpha+\beta)}{4}-\frac{(4b-\eta+\delta)}{4} \\ &= (a-b)-\frac{(\alpha-\beta-\eta+\delta)}{4} \end{aligned} \tag{10}$$

3. Fuzzy Linear Regression Analysis Where the Dependent Variable is Fuzzy

Regression analysis is an area of statistics that deals with the investigation of the dependence of a variable upon one or more variables. The aim of the regression analysis is to estimate the unknown parameters of the model which is given by the relationship between the dependent and independent variables. Recently, much research has studied the fuzzy estimation. There are some approach exist in the literature for the estimation of the fuzzy regression model.

Two of them are frequently used in parameter estimation, one of which is proposed by Tanaka et al [21]. and known as linear programming approach and the other is fuzzy least square approach. When the $X_p = (x_{p1},...,x_{pn})$ is an n-dimensional non-fuzzy input and Y_p is the corresponding fuzzy output

The linear multivariate hybrid model relates the dependent variable to two or more independent variables as follows

$$\hat{\hat{Y}} = (a_0, c_{0,l}, c_{0,r}) + (a_1, c_{1,l}, c_{1,r})X_1 + (a_2, c_{2,l}, c_{2,r})X_2 + ... + (a_p, c_{p,l}, c_{p,r})X_p \tag{11}$$

Where p is the number of the independent variables, $(a_p, c_{p,l}, c_{p,r})$ is the p^{th} fuzzy slope coefficient, and $(a_0, c_{0,l}, c_{0,r})$ is the fuzzy intercept coefficient [4].

For a set of data $((X_{1,i}, X_{2,i},..., X_{p,i} : (Y_i, e_{i,l}, e_{i,r}), i = 1,...,n))$ the normal equations for the fuzzy centers are as follows;

$$\begin{aligned}
&na_0 + \left(\sum_{i=1}^{n} X_{1,i}\right)a_1 + \left(\sum_{i=1}^{n} X_{2,i}\right)a_2 + ... + \left(\sum_{i=1}^{n} X_{p,i}\right)a_p = \sum_{i=1}^{n} Y_i, \\
&\left(\sum_{i=1}^{n} X_{1,i}\right)a_0 + \left(\sum_{i=1}^{n} X_{1,i}^2\right)a_1 + \left(\sum_{i=1}^{n} X_{1,i}X_{2,i}\right)a_2 + ... + \left(\sum_{i=1}^{n} X_{1,i}X_{p,i}\right)a_p = \sum_{i=1}^{n} X_{1,i}Y_i, \\
&\vdots \\
&\left(\sum_{i=1}^{n} X_{p,i}\right)a_0 + \left(\sum_{i=1}^{n} X_{p,i}X_{1,i}\right)a_1 + \left(\sum_{i=1}^{n} X_{p,i}X_{2,i}\right)a_2 + ... + \left(\sum_{i=1}^{n} X_{p,i}^2\right)a_p = \sum_{i=1}^{n} X_{p,i}Y_i.
\end{aligned} \tag{12}$$

The normal equations for the left spreads and right spreads are similar and the normal equations for the fuzzy centers can be found in [4].

4. Fuzzy Inference Systems and ANFIS Architecture

4.1. Fuzzy Inference Systems

The fuzzy inference system forms a useful computing framework based on the concepts of the fuzzy set theory, fuzzy reasoning, and fuzzy if-then rules. The fuzzy inference system is a powerful function approximater. The basic structure of a fuzzy inference system consists of three conceptual components; a rule base, which contains a selection of the fuzzy rules, a database, which defines the membership functions used in the fuzzy rules, and a reasoning mechanism, which performs the inference procedure upon the rules to derive a reasonable output. There are several different types of fuzzy inference systems developed for the function approximation. In this study, the Sugeno fuzzy inference system, which was proposed by Takagi and Sugeno [20], will be used. When the input vector X is $(x_1, x_2,..., x_p)^T$, then the system output Y can be determinated by the Sugeno inference system as

R^L = If; ($x_1 = F_1^L$ and $x_2 = F_2^L$ and ... $x_p = F_p^L$)

Then; $Y = Y^L = c_0^L + c_1^L x_1 + ... + c_p^L x_p$ (13)

Where F_i^L is a fuzzy set associated with the input x_j in the L^{th} rule, Y^L is output due to rule R^L. The parameters used to define the membership functions for F_i^L is called as the premise parameters, and c_i^L are called as the consequence parameters. For a real-valued input vector $X = (x_1, x_2,..., x_p)^T$, the overall output of the Sugeno fuzzy inference systems a weighted average of the Y^L.

$$\hat{Y} = \frac{\sum_{L=1}^{m} w^L Y^L}{\sum_{L=1}^{m} w^L} \tag{14}$$

Where the weight w^L is the true value of the proposition $Y = Y^L$ and is defined as

$$w^L = \prod_{i=1}^{p} \mu_{F_i^L}\left(x_j\right) \tag{15}$$

Where $\mu_{F_i^L}(x_j)$ is a membership function defined on the fuzzy set F_i^L.

4.2. ANFIS Architecture

Neural networks enabling the use of fuzzy inference system for prediction is known as the adaptive network. The ANFIS is a neural network architecture that can solve any function approximation problem. An adaptive network is a multilayer feed forward network in which each node performs a particular function on the incoming signals as well as the set of parameters pertaining to this node and it has five layers [12-14]. The formulas for the node functions may vary from node to node and the choice of each node function depends on the overall input-output function which the adaptive network is required to be carried out.

The fuzzy rule number of the system depends on the numbers of the independent variables and fuzzy sets numbers forming the independent variables. When an independent variable number is indicated with *p*, if the level number belonging to each variable is indicated with l_i $(i = 1, \ldots, p)$ the fuzzy rule number is indicated with

$$L = \prod_{i=1}^{p} l_i \tag{16}$$

To illustrate how a fuzzy inference system can be represented by ANFIS, let us consider the following example. Suppose a data set has a two-dimensional input $X = (x_1, x_2)$. For the input x_1, there are two fuzzy sets “C11” and “C12” and for the input x_2, two fuzzy set “C21” and “C22”. In this case a fuzzy inference system contains the following four rules:

$$\begin{aligned}
&R^1 : if\ (x_1\ is\ C11\ and\ x_2\ is\ C21), && then\ (Y^1 = c_0^1 + c_1^1 x_1 + c_2^1 x_2),\\
&R^2 : if\ (x_1\ is\ C11\ and\ x_2\ is\ C22), && then\ (Y^2 = c_0^2 + c_1^2 x_1 + c_2^2 x_2),\\
&R^3 : if\ (x_1\ is\ C12\ and\ x_2\ is\ C21), && then\ (Y^3 = c_0^3 + c_1^3 x_1 + c_2^3 x_2),\\
&R^4 : if\ (x_1\ is\ C12\ and\ x_2\ is\ C22), && then\ (Y^4 = c_0^4 + c_1^4 x_1 + c_2^4 x_2).
\end{aligned} \tag{17}$$

This fuzzy system is represented by the ANFIS as shown in Figure 2.

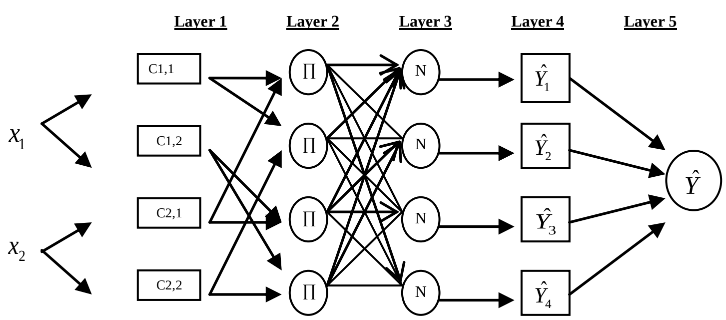

Figure 2: The ANFIS Architecture

The functions of each node in Figure 2 are defined as follows,

Layer 1: The output of node *h* in this layer is defined by the membership function on F_h

$$f_{1,h} = \mu_{F_h}(x_1), \quad for\ h = 1,2$$
$$f_{1,h} = \mu_{F_h}(x_2), \quad for\ h = 3,4 \tag{18}$$

Where the fuzzy cluster related to the fuzzy rules are indicated with $F_1, F_2, ..., F_h$ and μ_{F_h} is the membership function relates to F_h. Different membership functions can be defined for F_h. In this example, the Gaussian membership function will be used whose parameters can be represented by $\{\nu_h, \sigma_h\}$

$$\mu_{F_h}(x_1) = \exp\left[-\left(\frac{x_1 - \nu_h}{\sigma_h}\right)^2\right] \ for\ h = 1,2$$
$$\mu_{F_h}(x_2) = \exp\left[-\left(\frac{x_2 - \nu_h}{\sigma_h}\right)^2\right] \ for\ h = 3,4 \tag{19}$$

The parameter set $\{\nu_h, \sigma_h\}$ in this layer is referred to as the premise parameters.

Layer 2: Each nerve in the second layer has input signals coming from the first layer and they are defined as the multiplication of these input signals. This multiplied output forms the firing strength w^L for rule *L*:

$$f_{2,1} = w^1 = \mu_{F_1}(x_1) \cdot \mu_{F_3}(x_2),$$
$$f_{2,2} = w^2 = \mu_{F_1}(x_1) \cdot \mu_{F_4}(x_2),$$
$$f_{2,3} = w^3 = \mu_{F_2}(x_1) \cdot \mu_{F_3}(x_2),$$
$$f_{2,4} = w^4 = \mu_{F_2}(x_1) \cdot \mu_{F_4}(x_2). \tag{20}$$

Layer 3: The output of this layer is a normalization of the outputs of the second layer and nerve function is defined as

$$f_{3,L} = \overline{w}^L = \frac{w^L}{\sum_{L=1}^{4} w^L} \quad L = 1,...,4 \tag{21}$$

Layer 4: The output signals of the fourth layer are also connected to a function and this function is indicated with

$$f_{4,L} = \overline{w} Y^L \tag{22}$$

Where, Y^L stands for the conclusion part of the fuzzy if-then rule and it is indicated with

$$Y^L = c_0^L + c_1^L x_1 + c_2^L x_2 \tag{23}$$

Where c_i^L are the fuzzy numbers and stands for the posteriori parameters.

Layer 5: There is only one node which computes the overall output as the summation of all the incoming signals [3, 6]

$$f_{5,1} = \hat{Y} = \sum_{L=1}^{4} \overline{w}^L Y^L \tag{24}$$

5. Parameter Estimation by ANFIS in case where Outputs are Non-Symmetric Triangular Fuzzy Number

In forming a switching regression model, one of the most important points is that it necessary to determine how many cluster of independent variables there are.

In parameter estimation studies conducted via the adaptive networks, the class numbers of the data sets related to independent variables are determined intuitively initially. In this study we used the validity criterion to determine the optimal class number.

The adaptive network used to predict the unknown parameters of the regression model is based on the fuzzy if-then rules and fuzzy inference system which are defined in Section 4.1. Neurons, which form the networks, are characterized with parameter functions. Functional relationships between the dependent and independent variables in the process of the adaptive networks are modeled and estimated based on the models.

The aim of the fuzzy adaptive network is to obtain the model of the relationship between the input-output data couples. The process of determining the parameters for a regression model begins by determining the class numbers of the independent variables and priori parameters. The priori parameters are the characterizing class from which the data are derived. The parameter set $\{v_h, \sigma_h\}$ is referred to as the premise parameters. After that, the posteriori parameters associated with the regression model is determined. The updating of the posteriori parameters to obtain the best regression model is based on the error measure. The difference between the outputs (prediction) obtained from the network and the output, which is targeted, is described as an error measure. In this study, since the outputs are non-symmetric fuzzy numbers the error measure which is described in Section 2 and given by the Eq. (10) is used.

The algorithm of determining the switching regression model in cases where the outputs are non-symmetric fuzzy number, the following is proposed.

Step 1: Optimal class numbers of independent variables are determined by using a validity function

$$S_i = \frac{\frac{1}{n}\sum_{i=1}^{l_i}\sum_{j=1}^{n}\mu_{ij}^m \left\| v_i - x_j \right\|^2}{\min_{i \neq j} \left\| v_i - v_j \right\|^2} \tag{25}$$

Note that $\|.\|$ is the usual Euclidean norm. In Eq. (25) μ_{ij} is the fuzzy membership degree of belonging to i^{th} cluster of j-observation and m>1 is the fuzziness index. When the lowest S_i value is observed, class number (l_i) with the lowest S_i value is defined as an optimal class number [22].

Step 2: A priori parameter set is determined. Spreading is determined intuitively according to the space in which the input variables gain value and to the class numbers of independent variables. Centre parameters are indicated by

$$v_i = \min(X_i) + \frac{\max(X_i) - \min(X_i)}{(l_i - 1)}(i-1) \qquad i = 1, 2, ..., p \tag{26}$$

Step 3: $\overline{w}^L$ weights are counted by using the Eq. (21) used to form matrix B to be used in counting posteriori parameter set.

Step 4: On the condition that dependent variable consists of non-symmetric fuzzy numbers, a posteriori parameter set is obtained as $c_i^L = (a_i^L, c_{i,l}^L, c_{i,r}^L)$. In this case, to determine posteriori parameter set, Eq. (27) is used

$$\tilde{Z} = (B^T B)^{-1} B^T \tilde{Y} \tag{27}$$

Here $\tilde{Y}, \tilde{Z}$ and B defined as

$$\tilde{Y} = [\tilde{y}_1, \tilde{y}_2, ..., \tilde{y}_n]^T \tag{28}$$

Here $\tilde{y}_i = (y_i, \alpha_i, \beta_i)$

$$\tilde{Z} = [(a_0^1, c_{0,l}^1, c_{0,r}^1), ..., (a_0^m, c_{0,l}^m, c_{0,r}^m), (a_1^1, c_{1,l}^1, c_{1,r}^1), ..., (a_1^m, c_{1,l}^m, c_{1,r}^m), ..., (a_p^1, c_{p,l}^1, c_{p,r}^1), ..., (a_p^m, c_{p,l}^m, c_{p,r}^m)]^T \tag{29}$$

and

$$B=\begin{bmatrix} \overline{w}_1^1, & \cdots, & \overline{w}_1^m, & \overline{w}_1^1 x_{11}, & \cdots, & \overline{w}_1^m x_{11}, & \cdots, & \overline{w}_1^1 x_{p1}, & \cdots, & \overline{w}_1^m x_{p1} \\ \vdots & & & & & & \overline{w}_k^l x_{jk} & & & \vdots \\ \overline{w}_n^1 & \cdots, & \overline{w}_n^m, & \overline{w}_n^1 x_{1n}, & \cdots, & \overline{w}_n^m x_{1n}, & \cdots, & \overline{w}_n^1 x_{pn}, & \cdots, & \overline{w}_n^m x_{pn} \end{bmatrix} \tag{30}$$

Step 5: By using a posteriori parameter set $c_i^L=(a_i^L,c_{i,l}^L,c_{i,r}^L)$, the switching regression model indicated by

$$\tilde{Y}^L=(a_0^L,c_{0,l}^L,c_{0,r}^L)+(a_1^L,c_{1,l}^L,c_{1,r}^L)X_1+...+(a_p^L,c_{p,l}^L,c_{p,r}^L)X_p \tag{31}$$

And, the prediction values are obtained using

$$\hat{\tilde{Y}}=\sum_{L=1}^{m}\overline{w}^L\tilde{Y}^L \tag{32}$$

Step 6: Error related to the model is counted as

$$\varepsilon=\frac{1}{n}\sum_{i=1}^{n}[d(\tilde{y}_i-\hat{\tilde{y}}_i)]^2=\frac{1}{n}\sum_{i=1}^{n}(e_i)^2 \tag{33}$$

In here $\tilde{y}_i=(y_i,\alpha_i,\beta_i)$ and $\hat{\tilde{y}}_i=(\hat{y}_i,\hat{\alpha}_i,\hat{\beta}_i)$. Difference between these two non-symmetric fuzzy numbers is calculated by using the Eq. (12) such as

$$e_i=d(\tilde{y}_i-\hat{\tilde{y}}_i)=(y_i-\hat{y}_i)-\frac{(\alpha_i-\beta_i-\hat{\alpha}_i+\hat{\beta}_i)}{4} \tag{34}$$

If $\varepsilon<\phi$, then posteriori parameter has been obtained as parameters of the regression models to be formed, the process is concluded. If $\varepsilon\geq\phi$, then Step 7 begins.

Here ϕ, is a law stable value determined by the decision maker.

Step 7: Central priori parameters specified in Step l, are updated with

$$v_i^{'}=v_i\pm t \tag{35}$$

In a way that it increases from the lowest value to the highest and decreases from the highest value to the lowest. Here, *t* is the size of the step;

$$t=\frac{\max(x_{ji})-\min(x_{ji})}{a} \quad j=1,...,n \;\; i=1,...,p \,. \tag{36}$$

And *a* is a stable value which is determinant of the size of the step and therefore iteration number.

Step 8: Estimation for each priori parameter obtained by change and error criterion related to these predictions are counted. The lowest of the error criterion is defined. Priori parameters giving the lowest error specified, and prediction obtained via the models related to these parameters is taken as the output.

In this work, it is considered that the dependent variable is consisting of non-symmetric fuzzy number. Using the program, which was coded in the MATLAB for proposed algorithm, the value of the prime parameters can change minimally, and the optimal value of the prime parameters can be selected by this program.

6. Numerical Example

The values related to the data set with one non-symmetric triangular fuzzy dependent variable and two crisp independent variables are displayed in Table 1. The data shown in Table 1, except β_i, are taken from Cheng and Lee (1999). The values of β_i are assumed by the authors. The prediction values derived from the adaptive neural network (ANN), which are related to the data set and the errors these predictions, are displayed in Table 1. In addition, the predictions that are obtained with hybrid fuzzy least-squares regression (HFLS) and errors related to these predictions are also displayed in the same table.

Table 1: Data Set

i	x_{1i}	x_{2i}	$\tilde{y}_i(y_i, \alpha_i, \beta_i)$	$\hat{\tilde{y}}_{i(ANN)}(\hat{y}_i, \hat{\alpha}_i, \hat{\beta}_i)$	$e_{i(ANN)}$	$\hat{\tilde{y}}_{i(HFLS)}(\hat{y}_i, \hat{\alpha}_i, \hat{\beta}_i)$	$e_{i(HFLS)}$
1	6.8590	9.6880	(4.9920, 1.2480, 1.0480)	(5.0013, 1.2482, 1.0481)	-0.0093	(4.8535, 1.2076, 1.0447)	0.1293
2	7.2150	0.6170	(4.8490, 1.2120, 1.1120)	(5.0725, 1.2121, 1.1121)	-0.2235	(4.9537, 1.2322, 1.0981)	-0.0962
3	3.1990	2.8980	(5.2560, 1.3140, 1.1140)	(4.8163, 1.3142, 1.1141)	0.4397	(4.9132, 1.2256, 1.0855)	0.3278
4	0.2600	9.1510	(4.9940, 1.2480, 1.1480)	(5.0853, 1.2482, 1.1482)	-0.0913	(4.8336, 1.2084, 1.0492)	0.1752
5	5.5280	7.1140	(3.2750, 0.8190, 0.6190)	(4.5576, 0.8190, 0.6191)	-1.2827	(4.8763, 1.2144, 1.0601)	-1.6127
6	3.5390	2.7660	(4.8370, 1.2090, 1.1090)	(4.8146, 1.2091, 1.1091)	0.0224	(4.9160, 1.2260, 1.0862)	-0.0690
7	9.4760	8.4430	(5.0420, 1.2600, 1.0600)	(5.3053, 1.2602, 1.0601)	-0.2633	(4.8772, 1.2113, 1.0515)	0.1547
8	5.9680	9.4460	(5.2760, 1.3190, 1.1190)	(4.8535, 1.3192, 1.1191)	0.4225	(4.8526, 1.2082, 1.0463)	0.4139
9	7.5620	2.6780	(5.3840, 1.3460, 1.1460)	(4.8789, 1.3462, 1.1461)	0.5051	(4.9326, 1.2266, 1.0859)	0.4366
10	5.1030	3.3240	(4.3790, 1.0950, 0.9950)	(4.7154, 1.0951, 0.9951)	-0.3364	(4.9160, 1.2246, 1.0826)	-0.5265
11	2.8020	6.1010	(4.2080, 1.0520, 0.8520)	(4.7613, 1.0521, 0.8521)	-0.5533	(4.8767, 1.2169, 1.0666)	-0.6812
12	2.8310	6.4920	(4.8870, 1.2220, 1.1220)	(4.7728, 1.2222, 1.1221)	0.1142	(4.8726, 1.2159, 1.0643)	0.0273
13	8.2870	6.0810	(5.1670, 1.2920, 1.0920)	(4.7066, 1.2922, 1.0921)	0.4604	(4.8983, 1.2175, 1.0657)	0.2566
14	5.4790	2.9990	(3.3820, 0.8450, 0.7450)	(4.7530, 0.8450, 0.7451)	-1.3710	(4.9210, 1.2256, 1.0844)	-1.5287
15	0.4230	0.8850	(5.0330, 1.2580, 1.0580)	(5.1236, 1.2582, 1.0581)	-0.0906	(4.9243, 1.2308, 1.0979)	0.0919
16	6.1340	2.8240	(4.2730, 1.0680, 0.9680)	(4.7869, 1.0681, 0.9681)	-0.5139	(4.9254, 1.2261, 1.0853)	-0.6422
17	8.9760	1.1650	(5.1600, 1.2900, 1.0900)	(5.1253, 1.2902, 1.0901)	0.0347	(4.9546, 1.2309, 1.0945)	0.1895
18	2.3160	7.1210	(5.3100, 1.3280, 1.2280)	(4.8611, 1.3282, 1.2282)	0.4489	(4.8637, 1.2141, 1.0607)	0.4596
19	0.0800	2.1270	(5.0360, 1.2590, 1.0590)	(5.0002, 1.2592, 1.0591)	0.0358	(4.9094, 1.2274, 1.0906)	0.1108
20	2.9370	1.3000	(4.7550, 1.1890, 1.0890)	(4.9882, 1.1891, 1.0891)	-0.2332	(4.9296, 1.2299, 1.0949)	-0.1658
21	5.4080	1.3430	(6.0470, 1.5120, 1.3120)	(4.9527, 1.5122, 1.3122)	1.0943	(4.9388, 1.2300, 1.0942)	1.0922
22	6.5120	6.0410	(5.1630, 1.2910, 1.1910)	(4.5277, 1.2912, 1.1912)	0.6354	(4.8918, 1.2174, 1.0663)	0.2839
23	3.5040	5.5340	(5.4090, 1.3520, 1.1520)	(4.6763, 1.3522, 1.1522)	0.7327	(4.8856, 1.2185, 1.0698)	0.5105
24	9.3190	1.5160	(5.0750, 1.2690, 1.1690)	(5.1261, 1.2692, 1.1692)	-0.0511	(4.9521, 1.2299, 1.0924)	0.1323
25	6.8790	2.9060	(5.3180, 1.3300, 1.2300)	(4.8076, 1.3302, 1.2302)	0.5104	(4.9275, 1.2259, 1.0847)	0.4009
26	6.7930	0.1420	(5.0350, 1.2590, 1.1590)	(5.0952, 1.2592, 1.1592)	-0.0602	(4.9572, 1.2334, 1.1010)	0.0859
27	8.3250	2.4320	(4.9040, 1.2260, 1.0260)	(4.9659, 1.2262, 1.0261)	-0.0619	(4.9383, 1.2274, 1.0872)	-0.0492
28	0.5390	8.2110	(5.0120, 1.2530, 1.1530)	(5.0513, 1.2532, 1.1532)	-0.0393	(4.8449, 1.2110, 1.0546)	0.1812
29	1.5440	6.9000	(4.8630, 1.2160, 1.0160)	(4.9238, 1.2161, 1.0161)	-0.0608	(4.8631, 1.2146, 1.0622)	-0.0120
30	9.2980	2.5660	(4.8260, 1.2070, 1.1070)	(5.0410, 1.2071, 1.1071)	-0.2150	(4.9406, 1.2271, 1.0862)	-0.1044
ERROR				$\varepsilon_{(NN)} = 0.2638$		$\varepsilon_{(HFLS)} = 0.2925$	

The unknown coefficients of the hybrid fuzzy least-squares regression can be obtained by following three sets of simultaneous equations which are derived from the Eq. (12).

$$\begin{aligned} 30a_0 + 153.0950a_1 + 130.8110a_2 &= 147.1470 \\ 153.0950a_0 + 130.8110a_1 + 616.4527a_2 &= 752.446 \\ 130.8110a_0 + 616.4527a_1 + 822.9631a_2 &= 638.6706 \end{aligned} \quad (37)$$

$$\begin{aligned} 30c_{0,l} + 153.0950c_{1,l} + 130.8110c_{2,l} &= 36.788 \\ 153.0950c_{0,l} + 130.8110c_{1,l} + 616.4527c_{2,l} &= 188.1215 \\ 130.8110c_{0,l} + 616.4527c_{1,l} + 822.9631c_{2,l} &= 159.6706 \end{aligned} \quad (38)$$

$$\begin{aligned} 30c_{0,r} + 153.0950c_{1,r} + 130.8110c_{2,r} &= 32.2880 \\ 153.0950c_{0,r} + 130.8110c_{1,r} + 616.4527c_{2,r} &= 165.0179 \\ 130.8110c_{0,r} + 616.4527c_{1,r} + 822.9631c_{2,r} &= 139.3060 \end{aligned} \quad (39)$$

By solving the simultaneous equations sets (37),(38) and (39) we obtained respectively

$$a_0 = 4.9323,\ \ a_1 = 0.0039,\ \ a_2 = -0.0109$$

$$c_{0,l} = 1.2331,\ \ c_{1,l} = 0.0001,\ \ c_{2,l} = -0.00272$$

$$c_{0,r} = 1.1032,\ \ c_{1,r} = -0.00022,\ \ c_{2,r} = -0.00591$$

Therefore, a hybrid fuzzy least-square regression model is given by

$$\hat{\hat{Y}} = (4.9323, 1.2331, 1.1032) + (0.0039, 0.0001, -0.0002)X_1 + (-0.0109, -0.0027, -0.0059)X_2 \quad (40)$$

The differences between the target outputs and the predictions obtained from the hybrid fuzzy least-square regression model, located in Eq. (34), are given by $e_{i(HFLS)}$ which are obtained from the Eq. (24) and the error value is calculated as $\varepsilon_{(HFLS)} = 0.2925$ by using the Eq. (33).

From the beginning step of the proposed algorithm, for every two independent variables the fuzzy class numbers are determined as *l=2*. The number of the fuzzy inference rules formed according to this indicated the fuzzy class numbers is defined as four by using the Eq. (16).

The models that are obtained from the four fuzzy inference rules to non-symmetric triangular fuzzy output are

$$\begin{aligned}
\hat{\hat{Y}}_1 &= (5.1741, 1.2933, 1.1426) + (-0.0367, -0.0088, 0.0383)X_1 + (-0.2175, -0.0555, -0.0598)X_2 \\
\hat{\hat{Y}}_2 &= (6.7164, 1.6913, 1.1692) + (0.1271, 0.0311, 0.0490)X_1 + (-0.1093 - 0.0286, 0.0249)X_2 \\
\hat{\hat{Y}}_3 &= (5.4710, 1.3632, 1.0739) + (0.0115, 0.0033, 0.0109)X_1 + (0.1200, 0.0302, 0.0244)X_2 \\
\hat{\hat{Y}}_4 &= (-2.4836, -0.6147, -0.7098) + (0.4185, 0.1038, 0.1238)X_1 + (0.4082, 0.1022, 0.0630)X_2
\end{aligned} \quad (41)$$

The differences between the target outputs and the predictions are given by $e_{i(ANN)}$. The predictions are obtained from models in Eq. (41) which are obtained from proposed algorithm. These differences are also obtained from the Eq. (34) and the error value is calculated as $\varepsilon_{(ANN)} = 0.2638$ by using the Eq. (33). Errors from both methods are shown in Figure 3. Figure 3-a. shows errors which are obtained from ANN and Figure 3-b. shows errors which are obtained from HFLS.

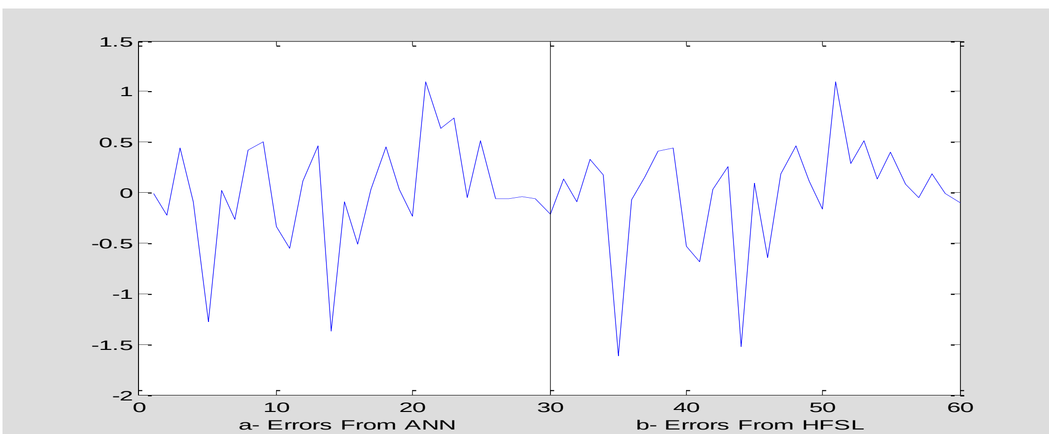

Figure 3: Errors from ANN and HFSL

7. Conclusion

Different methodologies have been developed for the estimating of the fuzzy regression model. In this study it is considered that the dependent variable consists of non-symmetric fuzzy numbers. Since the target outputs and the prediction values are non-symmetric fuzzy number, we used the fuzzy ranking method to determinate the error measure. Through numerical examples, we realize that the proposed algorithm for fuzzy regression model based on fuzzy inference system derive the satisfying solutions. This algorithm works well for the data with non-symmetric output. The prediction values obtained from the proposed algorithm are compared whit the prediction values obtained from the hybrid fuzzy least-squares regression method which is proposed by Chang (2001). Both methods are suitable for the prediction the multiple regression models in case the outputs are non-symmetric fuzzy numbers. According to the indicated error criterion, the errors related to the predictions that are obtained from the proposed algorithm are less than the errors obtained from the hybrid fuzzy least-squares regression method.

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