Estimation Bootstrap of the Probability Distributions of the Loss Function of Taguchi and of the Signal-To-Noise Ratio

Marcos Augusto Mendes Marques
FAE-Centro Universitário/PPGMNE-UFPR
Universidade Federal do Paraná
Brazil

Anselmo Chaves Neto
Statistics Department/PPGMNE-UFPR
Universidade Federal do Paraná
Brazil

Abstract

The knowledge of the probability distributions in experiments it is necessary for the accomplishment of statistical analyses of the experimental data. Such analyses have as objective to obtain relative information to the population of origin of the data supplied by random samples. This way, applied statistical inferences to the quality indicators as the loss function of Taguchi and signal to noise ratio should be used as a method for taking decisions regarding the quality of a product (service) and of the process of manufacture itself. Such indicators, together, measure the bad quality and robustness of a product in relation to variable answer of a functional characteristic. As it is necessary the knowledge of the probability distributions for the accomplishment of those inferences and, as in general, they are ignored is reasonable the application of the method intensive computationally of resample known like Bootstrap, for an estimate of those distributions.

Keywords: Quality, *Bootstrap*, Probability distributions, Statistical inference, Simulation.

1. Introduction

Investments in quality are necessary for a company to stay competitive in the market; however, it is necessary to justify such investments. The justification of those investments can be made through indicators that measure the quality or the bad quality of a product (service), from the project phase to its effective production. The loss function of Taguchi, the signal to noise ratio and the indicators of potential process capability are used to measure the quality loss and robustness of a product, being verified the same is in agreement with the project specifications and if its performance has suffered little influence of sources of external noises. According to Taguchi, quality can be defined as the suffered damage by the society as a consequence of the functional variation of the product (or service) and its adverse effects starting from the moment in that the product (or service) is received by the consumer (CHAVES NETO 2013). In agreement with the definition of Taguchi the direct application of the quality indicators can be verified. Due to the importance of the application of those indicators, it is convenient to accomplish statistical inferences in the same ones, with the intention of esteeming the population parameters of the population that it originated the data. The deduction of relative information to a population, by the use of random samples extracted from it, regards to the statistical inference (MARQUES; MARQUES 2009).

However, to accomplish statistical inferences, such as confidence intervals and tests of hypotheses, should know the probability distribution that generated the sample. The probability distributions of the loss function of Taguchi and of the signal to noise ratio are not known, therefore needs to esteem them. For the estimation of parameters that demand complex procedures of calculation or still, asymptotic methods are adapted to apply the solution Bootstrap. It is known a lot that Bootstrap is a computationally intensive method of resample introduced in 1979 by Bradley Efron (EFRON 1979). The advantage of the use of that method is the easy understanding and the generality with that can be applied.

The main proposal of this method is the use of intensive computing for the obtaining of standard error, confidence intervals, and probability distributions called Bootstrap distributions and of other measures of statistical uncertainties for a variety of problems.

"The resampling technique Bootstrap allows precisely to use a sample to estimate the amount of interest through a statistic and to also evaluate the properties of that statistic distribution, in other words, they also supply estimates for the distribution, bias, standard deviation and confidence intervals of the statistics" (LÚCIO, LEANDRO and PAULA 2006). The proposed objective in this work, it is the analysis of the application of the method Bootstrap for the estimate of the probability distributions for the loss function of Taguchi and for the signal to noise ratio.

2. Quality

The quality concept is not new. That notion had developed along the time, given to the specificities that each period presented in the history of the human development (PALADINI 1995). Even with the quality concept is not new and of developing in agreement with the historical period to, which is part, little understanding exists in its concept, in the academic and business world, and that difficulty is due to each person's fact to possess a personal concept of quality. Oakland (1994) he affirms that the quality concept depends on the singular's individual perception, in other words, what is considered of superior quality for some people cannot supply the needs of the other. This way, a quality product in the consumer's vision, is that who assists to its needs and that it is inside of its purchase possibility, in other words, have fair price (CSILLAG 1991). However, in agreement with the technical definition of quality, the quality is measured as a fraction of defective products, in other words, it’s considered defective to that product that is not in agreement with the technical specifications, not taking into account, at this time, the consumer's opinion regarding of its quality.

The technical definition of quality is enunciated as: "quality is a characteristic of the production process that should be measured by the proportion of goods or services produced that reach the specified properties of the project" (CHAVES NETO 2013). This definition considers only the parts involved in the productive process.
Taguchi considers incomplete this definition of quality and it is not accepts that all of the products that are inside of the specification limits, for a certain characteristic, possess the same quality. The quality definition, according to Taguchi, is the suffered damage to the society as a consequence of the functional variation of the product (or service) and its adverse effects starting from the moment of that the product (or service) is received by the consumer (CHAVES NETO 2013). Functional variation is defined as the deviation of the performance of the product, usually caused by noises regarding its nominal value (target) being this, the value wanted by the consumer for a better performance of the product (service).

The costs generated by products that assist the specifications, but that has a great functional variation (no uniformity regarding of some characteristics) will be detected only when the product is in the consumer's hands, generating costs of warranty activations repair of the product and so on. In agreement with the philosophy of Taguchi a product with superior quality is cheaper, with high reliability and solid performance, because the factors causes of the functional variation of some characteristic of it is minimized. When the influence of factors of external noises is minimized or eliminated to a product that it is called resulting product, the project is denominated of robust project. In the robust project, the objective is to adapt the product to the external variability conditions, in such a way that the acting of the same does not suffers great alterations for the action of such factors. How to measure the quality loss? How to verify a product it is robust? How to measure the process it is capable to produce inside of the certain specifications? The answer to those questions is in the quality indicators: loss function of Taguchi and signal to noise ratio. Applying the statistical techniques together with those indicators benefits the increase of the quality, the productivity, earnings and, consequently, of competitiveness of the companies and the countries.

3. *Loss Function of Taguchi*

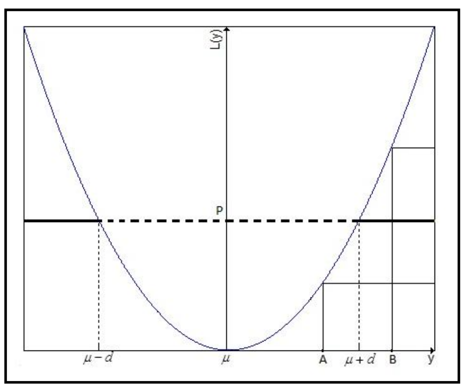
The knowledge of the costs involved in a productive process is in general of great importance for the industries and companies. As a result the cost of the absence of the quality can be decisive to the existence of the company. The continuous improvement of the product is effective in the reduction of the costs. In agreement with Taguchi, Elsayed and Hsiang (1990), the relationship between quality and price of a product are of extreme importance, because the price represents a loss for the consumer in the hour of the purchase and bad quality represents an additional loss during its use.

According to Almeira and Toledo (1989) this loss is defined as the expected value of the monetary loss caused by variances of the performance characteristics in relation to the wanted (or specified) value. Taguchi, apud Nakagawa (1993), it developed a method that allows to quantify the impact of the bad quality, in monetary units, through the loss function. As a result, it gets to measure the impact of the losses in a product not only for the consumer, but also for society in long period. Through the loss function, it is demonstrated that high quality is free of costs associated with lower quality.

Such statement is verified thoroughly in the Japanese and American industry, which through the loss function it was managed to quantify the benefits, obtained by the reduction of variability of a characteristic of the product around its target value. In agreement with Ealey (1988), with the use of the loss function, it is getting to justify investments in the improvement of the quality, being prioritized the development of the product and consequent balance between cost and quality. In relation to that philosophy it seeks to produce a product or service, minimizing that loss for the society, in other words, it will try to produce more uniform products that have a minimum functional variation. "Taguchi affirms that the loss for the society, as a whole, is minimum when the performance of the product meets to the nominal value and (wanted performance) and the quantification of this loss in monetary value does with that all understanding the importance of improvement of the production process for the product (service), from the simplest worker to the president of the company" (CHAVES NETO 2013). There are three types of loss functions where each one is in agreement with the corresponding functional characteristic. Therefore, we have the loss function nominal-the-better (NTB), loss function smaller-the-better (STB) and loss function larger-the-better (LTB). The figure 1 demonstrates the loss function quadratic of Taguchi for the nominal-the-better characteristic, and it represents the loss due to the functional variation of some variable answer in relation to the project nominal value. This loss function assumes its nominal value of projects in μ and the loss function $L(y)$ assumes the null value for $y = \mu$. The loss function, in this case, it is represented for:

$$L(y) = k(y - \mu)^2 \tag{1}$$

Figure1: Loss Function of Taguchi



Source: The author

The loss function (1), represented in the figure 1, it accuses an increase in the loss of measure that the variable y stands back of the target μ. The proportionality constant k, denominated of coefficient of quality loss, it represents the proportion of the financial importance of the customer in terms of the target value and its calculation is done starting from the relationship of the functional tolerance with the consumer's loss. The loss

Function smaller-the-better (STB), represented by the equation (2), it has as variable answer $y \in R^+$, being its nominal value the same as zero.

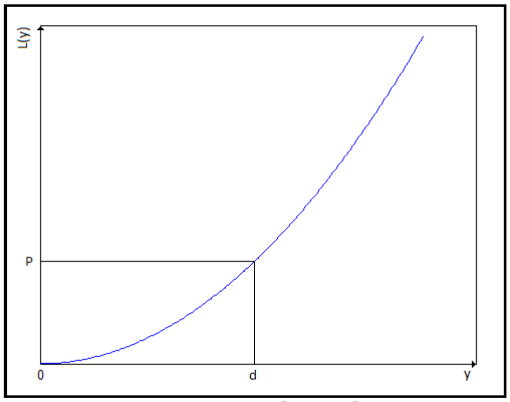
$$L(y) = ky^2 \tag{2}$$

The figure 2 represents the behavior of the loss function described in the equation (2). In a similar way, the loss function nominal-the-better, the proportionality constant k is denominated of coefficient of quality loss, and its value is calculated considering the consumer's loss in relation to tolerance functional for the analyzed functional characteristic. The loss function larger-the-better (LTB), represented by the equation (3), it has as variable answer $y \in R^+$, where the nominal value is the possible largest, in other words, it tends to infinity, because as larger this value smaller is the loss value for the society.

$$L(y) = k\left(\frac{1}{y^2}\right) \tag{3}$$

By the same as in the loss functions defined previously, k represents the coefficient of the consumer's loss.

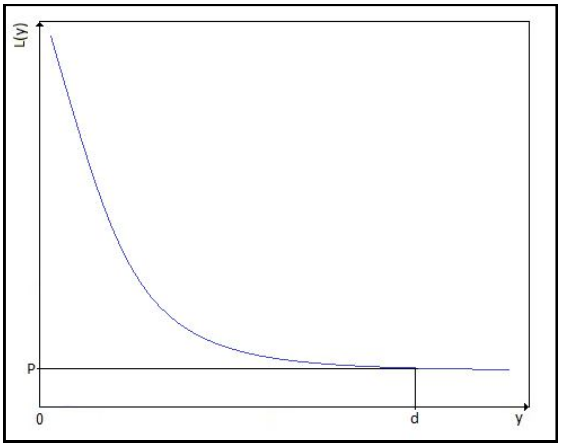
Figure 2: Loss Function Smaller-the-Better



Source: The author

The figure 3 demonstrates the behavior of the loss function, defined in the equation (3), this way, as mentioned previously, as larger the value than y assumes minor is the loss.

Figure 3: Loss Function Larger-the-Better



Source: The author

Frequently the manufacturers need to evaluate the average quality of its product, in relation to a characteristic of the product, along a period of time, and is measured through the development of the average loss function for n units of a product (service). The method employed to estimate this is an average at an arithmetic rate of the quadratic variances in relation to the nominal valueμ. It is verified that the average loss function for of the characteristics of the type nominal-the-better, smaller-the-better and larger-the-better, are given respectively by the equations (4), (5) and (6).

The average loss function for the nominal-the-better functional characteristic is defined as

$$\overline{L}(y) = k\left[\sigma^2 + (\bar{y} - \mu)^2\right] \tag{4}$$

The average loss function for the lower-the-better functional characteristic is defined as

$$\overline{L}(y) = k\left[\sigma^2 + \bar{y}^2\right] \tag{5}$$

and the average loss function for the larger-the-better functional characteristic is defined as

$$\overline{L}(y) = k\left(\frac{1}{\overline{y}^2}\right)\left[1 + \left(\frac{3\sigma^2}{\overline{y}^2}\right)\right] \tag{6}$$

Where σ^2 it is the relative variance to the average $\overline{y}$; $\overline{y}$ it is the value of the arithmetic average of y in the group sample and *k* is the coefficient of quality loss.

4. Signal to Noise Ratio

Taguchi et al. (2004) idealized a transformation of the data in the repetition of an experiment, in such a way that this transformation represents the measure of the existent variation and its influence in some functional characteristic of the product or process. This transformation is denominated of signal to noise ratio (S/N) and it combines several repetitions with the intention of verifying how much of functional variation it is present in the performance of the product. The signal to noise ratio is a robust measure, once it measures the transformation of energy in the product and it has as scale the decibel (dB). The signal to noise ratio measures the magnitude of the true information (signal) after some uncontrollable (noise) variations. In this case, it is called as signal the wanted performance and as noise the performance unwelcome. The procedure for construction of the signal to noise ratio, is based on the mean square deviation (MSD) of the average loss function of quality, which is calculated through dispersion measures, just like the variance. The mean square deviation is modified to do the signal to noise ratio independent of the adjustments of the target value of the project. There are four types of signal to noise ratio and each one of them is related to one of the three types of existent functional characteristics. For the nominal-the-better (NTB) functional characteristic, it has two types of signal to noise ratio. The first type, denominated of signal to noise ratio NTB type I, which is given by the equation

$$S/N_{tipo\ I-NTB} = 10\log\left[\frac{\overline{y}^2}{s^2}\right] \tag{7}$$

where

$$\overline{y} = \frac{\sum_{i=1}^{n} y_i}{n} \tag{8}$$

and

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(y_i - \overline{y}\right)^2 \tag{9}$$

In this type of signal to noise ratio, the variable is composed of real numbers no negative, in other words, the variable answer is defined as a continuous random variable no negative. The second type of signal to noise ratio, for the nominal-the-better characteristic, it is denominated of signal to noise ratio NTB type II, defined for

$$S/N_{tipo\ II-NTB} = -10\log\left[s^2\right] \tag{10}$$

Where s^2 is calculated by equation (9).

In this type of signal to noise ratio NTB the values assumed by the variable answer is the group of the real numbers, being one of the factors that differentiates it of the type I. Another factor that differentiates the two types NTB of signal to noise ratio is that, in this case, the target value can be the same to zero. However, for many performance characteristics that have the zero as target value, it becomes more convenient, it is modeling for the signal to noise ratio smaller-the-better (STB), which is the same for its estimate, it is based on the loss function smaller-the-better.

The signal to noise ratio STB is defined by

$$S/N_{STB} = -10\log\left[\frac{1}{n}\sum_{i=1}^{n} y_i^2\right] \tag{11}$$

Where y_i represents the value of the variable analyzed answer. The fourth type of signal to noise ratio, denominated of signal to noise ratio large-the-better (LTB) its bases on the loss function large-the-better, whose variable answer is composed of real numbers no negative, in other words $y \in R^+$. In this case, the variable answer is the possible largest or infinite. This signal to noise ratio is given for

$$S/N_{LTB} = -10\log\left[\frac{1}{n}\sum_{i=1}^{n}\frac{1}{y_i^2}\right] \quad (12)$$

Where y_i, in way similar to the case STB, represents the value of the variable analyzed answer.

In the equations regarding the signal to noise ratio described, the same ones are used of the logarithm by the fact of improving the addictively property, the property of reduction of the interaction among the control factors and also for the fact of the same if it uses of the scale decibel to relate two values. Therefore, as it does not know the probability distributions of the loss functions and of the signal to noise ratio described, the resampling method Bootstrap will be used for the estimate of the same ones.

5. Bootstrap Method

In several experiments, for a group of available data, the uncertainty of an estimate can be made analytically calculations based on a known model of probability. However, the obtaining of that estimate for more complex experiments for the uncertainty can be complicated and very extensive. Some measured in statistics are used thoroughly; being the arithmetic average and the standard deviation is the simplest statistics of obtaining in a clear way. However, for some complex statistics, there is not an available formula for the calculation of the same or, the way to obtain it is very complex if used the traditional ways in statistics. In this context is adapted the use denominated technique of resampling method Bootstrap. The advantage of using that resampling method, besides being of easy understanding, is the generality with that it can be applied, because it needs that few suppositions being made, supplying more necessary answers than other resampling methods. In agreement with Chaves Neto et al. (2014), the method Bootstrap allows the evaluation of the variability of any statistics, based on information on a single sample. The central proposal of the method Bootstrap is to demonstrate through the use computational the obtaining of standard error, confidence intervals, the estimated probability distribution and the calculation of other measures of statistical uncertainties for a variety of problems.

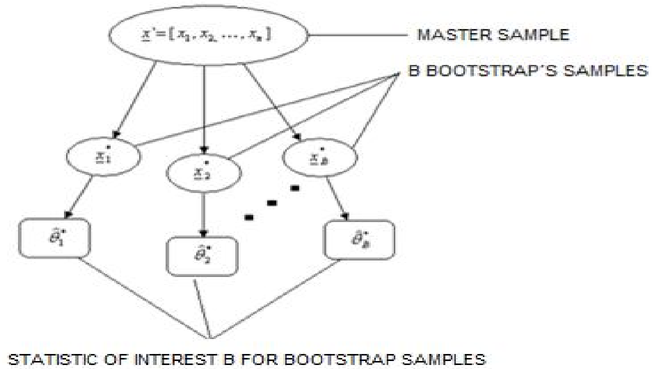
In 1979, Bradley Efron published its first article on the method Bootstrap, and this way, it revolutionized the resampling idea, because he synthesized some ideas previously of resampling and it established a new picture for statistical analyses based on simulations. In this new paradigm, the method Bootstrap was shown more efficient than the methods previously used. Due to its great success, such method, contributed and it still contributes a lot to productions of academic world. "Bootstrap is a statistical technique related to the jackknife, introduced by Efron in 1979 and it is shown versatility in the treatment of a variety of statistical problems" (CHAVES NETO 1986). Such method is very useful when it has a sample of small size, or when the estimate calculations for analytical means if they turn extremely complicated and even impossible of obtaining, because not always it is held in a clear way the probability distribution that generated the sample. In that case, bootstrap becomes extremely useful, because the same one can be used even if not knowing the original distribution that generated the statistical sample of the parameter to be studied. To accomplish the estimate of a parameter for bootstrap it becomes necessary a large resampling number, and for such it is necessary the assistance of a computers programs and fast computers. According to Davison and Hinkley (1997) as this approach consists of repeating the procedure of analyses of data with many replicating groups, this procedure is denominated of intensive computationally. This way the method is used a lot for estimation of sampling distributions.

The main objective in the use of Bootstrap as resampling method is to demonstrate, through the computer, the use of data for the obtaining of a reliable standard error, reliable confidence intervals and other reliable statistical measures for a range of problems. In order to use the method, in a simplified way, is made the resampling through the collected data of the experiment. On the top of that, a variety of problems in statistics can be avoided, such as excessive simplifications in more complex problems. To begin with method Bootstrap, should it take a master sample (original sample) of size n $\underline{x}' = [x_1, x_2, \ldots, x_n]$, that is representative of the population.

For such, this sample should be obtained in a planned way for not committing calculation mistakes and with that to obtain results did not reliably. The resampling of this sample master should be equal to many samples of the population of interest and the obtained distribution of a statistic of interest, through these resamples; it represents the distribution of an estimate sample probability of these statistics. Being B the numbers of samples Bootstrap (resamples), the most of authors recommend the use of B the same as at least 1000. These B samples should have the same size n $\underline{x}_1^*, \underline{x}_2^*, \ldots, \underline{x}_B^*$ of master sample, such that the chosen values starting from the master sample should be done in a random way and with replacement. Given the resamples, the statistics $\hat{\theta}_i^*$ should be calculated for each one of them. This way, according to Bussab and Morettin (2012) "the basic idea of Bootstrap is resampling the available group of data to estimate the parameter θ, in order to create replicated data. From those replications can be measured the variability of an estimated proposed for θ, without appealing to analytical calculations."

Then from those resamples can be calculated the statistics $\hat{\theta}_i^*$ of each interesting sample Bootstrap, where the overwritten "*" indicates that it has the calculation of a statistics starting from a sample Bootstrap. The sample master's statistics will be represented for $\hat{\theta}$. The calculation is based in calculating the statistics of interest for each of the "B samples Bootstrap", according to the figure 4.

Figure 4: Structure of *Bootstrap Method*



Source: The author

The process is based on the application of an algorithm to generate a number B of independents samples Bootstrap denominated of size n (same size that the master sample) denominated of $\underline{x}_1^*, \underline{x}_2^*, \ldots, \underline{x}_B^*$, and starting from those B samples it is considered the statistics of interest for each one of the B samples denominated estimates Bootstrap and denoted for $\hat{\theta}_1^*, \hat{\theta}_2^*, \ldots, \hat{\theta}_B^*$. From that calculation of $\hat{\theta}_1^*, \hat{\theta}_2^*, \ldots, \hat{\theta}_B^*$ it has the distribution of frequencies of those Bootstrap samples and the histogram correspondent the distribution Bootstrap of the statistics of interest. In this work the interest is in the average loss functions of Taguchi and signal to noise ratio.

6. Material and Methods

In this work the method Bootstrap was applied to a sample with smaller-the-better functional characteristic, whose values samples are removed from the book called Engineer of the Quality in Systems of Production, of authorship of Taguchi et al. (1990). The reason of using the data of this book is for the fact of, in practice, to be difficult the obtaining of the consumer's loss of certain functional tolerance, because it depends on direct and indirect costs The indirect costs, just as the customer's dissatisfaction is difficult to be measured. In the approached situation the variable of interest sample is the uniformity of the surface of a standard block, because the more uniform the surface, best will be the block. Its functional tolerance is defined in $12\ \mu m$ with a loss for the consumer in \$80. The supplied data are regarding the flatness data of blocks manufactured by two machines M_1 and M_2, according to table 1.

Table 1: Flatness Data of Machines 1 and 2

MACHINE	FLATNESS DATA (μm)									
M_1	0	5	4	2	3	1	7	6	8	4
	6	0	3	10	4	5	3	2	0	7
M_2	5	4	0	4	2	1	0	2	5	3
	2	1	3	0	2	4	1	6	2	1

Source: Taguchi, G. *et al*, 1990, p. 46

The samples are regarding of two machines. It is used of two samples (machines) to make a comparison regarding the measures of the average loss functions of Taguchi, signal to noise ratio and estimates of the distributions bootstrap of both. The resampling, the calculations of the average loss functions of Taguchi and of the signal to noise for each resample, as well as the plotting of the distributions Bootstrap were accomplished in the software Matlab. The average loss functions of Taguchi for each resample is given for

$$\hat{\bar{L}}_i^*(y) = 0{,}25\left[\sigma_i^2 + (\bar{y}_i - 115)^2\right] \tag{13}$$

Where $i = 1, \ldots, B$.

This way it has $\hat{\bar{L}}_1^*(y), \hat{\bar{L}}_2^*(y), \ldots, \hat{\bar{L}}_B^*(y)$ and starting from these estimates the distribution Bootstrap of the average loss functions of Taguchi is calculated for each one of the machines. The signal to noise ratio for each resample is given for

$$S/N_{i\ STB}^* = -10\log\left[\frac{1}{n}\sum_{j=1}^{n} y_{ij}^2\right] \tag{14}$$

Where $i = 1, \ldots, B$.

This way it has $S/N_1^*, S/N_2^*, \ldots, S/N_B^*$ and starting from these estimates the distribution Bootstrap of the signal to noise ratio is made calculations for each one of the machines. In this article, the distributions Bootstrap (probability distributions) are coming of B equal to 10000 samples Bootstrap, because for larger samples, the graphic characteristics of the distributions remain practically the same.

7. Results and Analysis

The distributions Bootstrap obtained regarding the average loss functions of Taguchi and signal to noise ratio for 10000 samples Bootstrap are represented in the graphs 1 and 2. The reason to choose B equal to 10000 is for the fact of the variability, for samples with B bigger than 10000, to be the minimum, according to lists 2 and 3. The table 2 demonstrates the results regarding average loss functions of Taguchi, for 10000, 20000 and 30000 samples Bootstrap and it is observed that the variability measured by the standard error Bootstrap (standard deviation Bootstrap) varies too little when it increases the size of the samples Bootstrap, justifying the choice of 10000 resampling as enough to estimate of the distribution of probability of the average loss functions of Taguchi.

Table 2: Results for the Average Loss Function to the Case STB

CALCULATION	M_1			M_2		
	B			B		
	10000	20000	30000	10000	20000	30000
$\bar{L}(y)$	13,0000	13,0000	13,0000	4,8889	4,8889	4,8889
SE_{boot}	3,1792	3,1197	3,1556	1,2485	1,2481	1,2425

Source: The author

The table 3 demonstrates the results regarding the signal to noise for 10000, 20000 and 30000 samples Bootstrap and analogously to the table 2, it is verified that 10000 resamples are enough to estimate of the probability distribution of the signal to noise ratio.

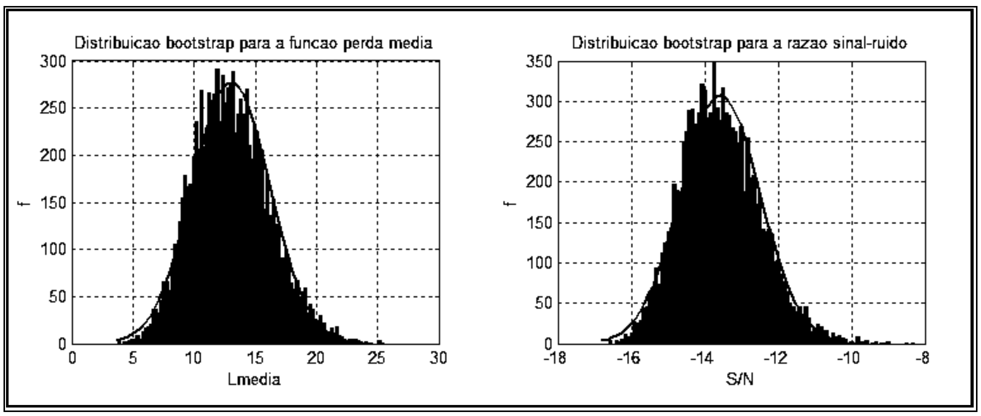
Table 3: Results for Signal to Noise Ratio to the Case STB

CALCULATION	M_1			M_2		
	B			B		
	10000	20000	30000	10000	20000	30000
S/N (dB)	-13,6922	-13,6922	-13,6922	-9,4448	-9,4448	-9,4448
SE_{boot} (dB)	1,0956	1,0740	1,0918	1,1678	1,1619	1,1554

Source: The author

The graph 1 display the distributions of estimate probabilities of the average loss function of Taguchi and of the signal to noise ratio for the machine 1, whose functional characteristic of the smaller-the-better type is represented by the flatness data of a block.

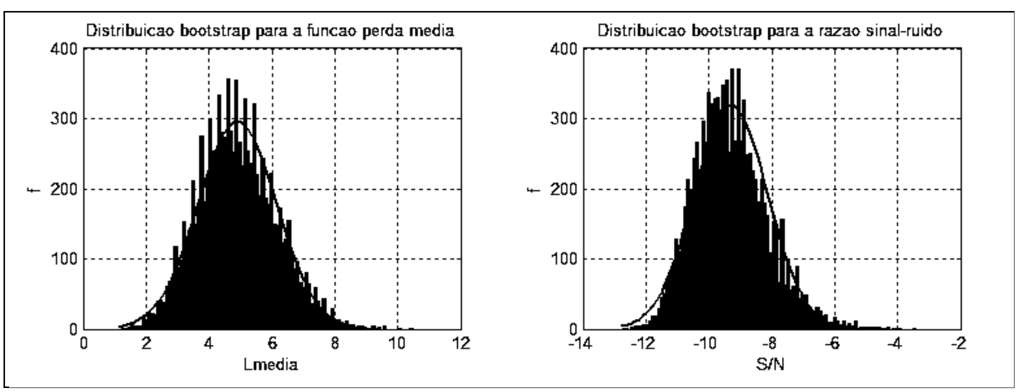
Graph 1: Distributions *Bootstrap* of M_1 to B = 10000



Source: The author

The graph 2 represents the distributions of estimate probabilities of the average loss function of Taguchi and of the signal to noise ratio regarding the flatness data production of blocks of the machine 2. It can be verified, jointly, for the tables 2 and 3 and for the graphs 1 and 2 that the machine 2, has a smaller average loss and a larger signal to noise ratio than the machine 1, characterizing for the philosophy of Taguchi that the blocks produced by the machine 2 has a superior quality that the one produced by the machine 1.

Graph 2: Distributions *Bootstrap* of M_2 to B = 10000



Source: The author

It is verified, also, certain similarity regarding the graphic format of the distributions Bootstrap for the sample referent machines 1 and 2, it is observing certain asymmetry in the distribution of estimate probability of the signal to noise ratio for both machines. One of the main objectives of knowing the probability distributions of both samples, regarding the average loss function of Taguchi and the signal to noise ratio is the accomplishment of statistical inferences in both samples, just as the calculation of confidence intervals.

8. Conclusion

The application of the method was shown efficient in the estimate of the probabilities distributions (distributions Bootstrap) of the average loss function of Taguchi and signal to noise ratio, where both were unknown. Together with the estimates of the probabilities distributions, it was calculated the punctual values of the average loss function of Taguchi and of the signal to noise ratio to the master sample of the machines M1 and M2, with the intention of comparing the quality of production of both machines. Due to the efficiency of the application for the method Bootstrap in the estimate of the probabilities distributions proposed, can extend its application to the other two types of remaining functional characteristics, being as a suggestion, also, the estimate of confidence intervals of the average loss function of Taguchi, signal to noise ratio and others quality indicators, such as the potential process capability, denominated of C_p, C_{pk} e C_{pkm}.

References

Bussab, W. de O.; Morettin, P. A. **Estatística básica**. 7ed. São Paulo: Saraiva, 2012.

Chaves Neto, A. **Bootstrap e regressão, uma opção aos mínimos quadrados generalizados**. Dissertação (Mestrado) – Unicamp/SP, 1986.

Chaves Neto, A. **Técnicas estatísticas aplicadas à engenharia de qualidade.** UFPR-PPGMNE. Curitiba: 2013.

Chaves Neto, A; Souza, R. C.; Chaves, I. A. **Structural Identification of Quasi White Noise Linear Models.** International Journal of Applied Science and Technology Vol. 4, No. 4; July 2014.

Davison, A. C.; Hinkley, D. V. **Bootstrap Methods and Their Application**. Cambridge: Univ. Press, 1997.

Efron, B. **Bootstrap methods: Another look at the Jackknife.** The Annals of Statistics: 1-26. 1979.

Lucio, P.S.; Leandro, I. V.; Paula, T. P. ***Bootstrap* aplicado à avaliação de incertezas estatísticas no prognóstico de quantis extremos de precipitação**. Instituto Nacional de Meteorologia (INMET) – Coordenação de Desenvolvimento e Pesquisa (CDP). Brasília, 2006. [Online; acesso em 06-Junho-2014]. Disponível em: > http://www.cbmet.com/cbm-files/14 eee1d97e2f1534538b995b911f9082b6.pdf.

Marques, J.; Marques, M. A. M. **Estatística básica para os cursos de engenharia.** 2 ed. Curitiba: Domínio do Saber, 2009.

Nakagawa, M. **Gestão estratégica de custos: conceito, sistemas e implementação.** São Paulo: Atlas, 1993.

Oakland, J. S. **Gerenciamento da qualidade total.** São Paulo: Nobel, 1994.

Paladini, E. P. **Gestão de qualidade de um processo: A qualidade na produção de bens e serviços.** São Paulo: Atlas, 1995.

Taguchi, G. et al. **Engenharia de qualidade em sistemas de produção.** Tradução Regina Cláudia Loverri; reavisão técnica: José Carlos de Castro Waeny. São Paulo: Mcgraw-Hill, 1990.

Taguchi, G.; Jugulum, R.; Taguchi, S. **Computer-Based robust engineering: essentials for DFSS.**Milwaukee: Quality Press, 2004.