

Modeling the Deviations of the Reflectorless Distance Measurement due to the Laser Beam's Incident Angle

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Abstract

The reflectorless (RL) distance measurements provide exceptional possibilities to the modern surveying projects, so the reliability of the measured distances has major significance. However the reflectorless distance measurement is influenced by numerous parameters via a complex interaction, which add extra error to the measured value. Several studies were carried out in order to determine the magnitude of these errors by using different instruments. This study investigates the determination of a correction equation for the error in the reflectorless distance measurements that occurs outdoors at a distance of 100m due to the change of the incident angle of the laser beam. Three advanced total stations, 23 materials and 13 different incident angles are participated in the experiments for the data collection. Deviations up to 5cm are registered. Numerous of approximation equations are tested in order to estimate, which describes better the deviation of the distance from its correct value. The RMSE and R^2 are used as criteria in order to ensure the reliability of the adaptations. It was found that each total station has a different behaviour. Thus one general correction equation of 2nd order is determined, for each total station and for all the involved materials, which corrects all the measurements by an accuracy of $\pm 3\text{mm}$. Moreover individual equations of 1st, 2nd, 3rd and exponential order are determined for each material. The results certify that the modelling of the deviations of the reflectorless distance measurement due to the beam's incident angle is feasible. This fact strengthens the possibility to incorporate such equations in total stations' software in order to automatically correct their reflectorless distance measurements.

Keywords: Distance measurement, Reflectorless mode, modern total station, incident angle, laser beam, approximation.

1. Introduction

About three decades now the reflectorless (RL) total stations boost the potentialities of Surveyors. The RL distance measurement open enormous fields of activities and applications that Surveyors couldn't ever imagine (Hope, CJ & Dawe, SW 2015, McCormac J, 1995). This laser technology makes feasible the distance measurement to inaccessible points. This capability upgrades mainly the monuments documentation as well as the monitoring of modern technical constructions (such as bridges, high buildings, dams etc.). Some advantages of the reflectorless distance measurement are the elimination of the time and labor needed for a work, the restrain of the crew to one person and the easy and accurate targeting of detail points as no prism is needed to be placed.

However some serious issues have been raised and discussed by the scientific community relative to the precision and the deviation of the RL distance measurement in comparison to the conventional one by using prisms (Khalil, R. 2015, Reda A. & Bedada B., 2012).

The size of the measured distance, the kind, the reflection, the texture (smooth or rough) and the color of the target, the shape and the size of the beam's footprint, the size of the target surface, the position of the measured point (recess or overhang), the measurement method (phase shift, time of flight or combination), the laser class which is used, the illumination of the environment, the atmospheric conditions and the incident angle of the laser beam in the surface are parameters that influence the final value of the RL measured distance. Many studies have been elaborated on this subject in order to investigate these parameters together or one by one (Ashraf A.A. et al 2011, Cameron 2003, Coaker L., 2009, Hossam 2015, Kowalczyk et al 2014, Mazalová 2010, Lambrou et al. 2010). Distance measurement mainly depends on the percentage of the beam that returns to the total station.

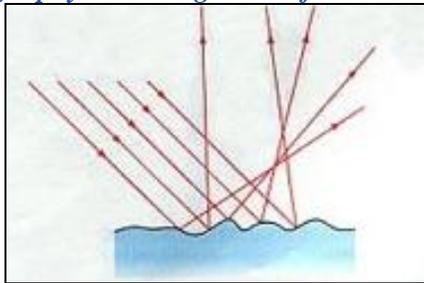
Only a small percentage of the laser beam is exploitable by the total station as almost the half of the emitted signal is vanished due to the absorption from dust, air molecules and water drops – the so called extinction and from the rest returned signal only a fraction is received by the EDM optics (Junyu M et al 2009). There is a minimum indispensable quantity of returnable beam, for a distance measurement to be succeeded. So, it is important the emitted EDM signal to be reflected back to the EDM receiver from the right point of the sighted surface.

Also, the atmospheric conditions is a parameter that influence the electromagnetic distance measurement with (Rüeger J. M 1996) or without prisms. For this reason temperature, pressure and humidity are measured in order to be inserted in the total station's computer, where the appropriate corrections are applied on the measurement in real time. Also snow, fog, rain and dust as well as the strong sunlight diminish the visibility and they limit the measurement possibilities (McCormac J., 1995). The outdoors measurements are influenced more by the atmospheric conditions than the indoors ones as the above parameters are changing more.

Apart from the atmospheric conditions other parameters that influence the RL distance measurement, are due to the physical characteristics of the surface, to the laser beam and to the geometry of the specific measurement.

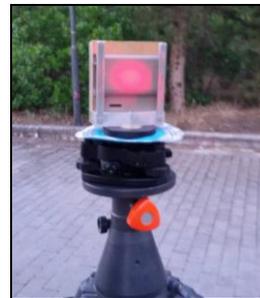
The case of the reflectorless distance measurement is a diffuse reflection as the pulse reflected in all directions. According to Lambert law as the beam incidents on a rough surface, the reflected light rays are scattered (figure 1a). The diffuse reflection of the beam leads to the dispersion of the waves' flight and so to a confused and unreliable measured distance (Reda A. & Bedada B., 2012). Also some multipaths that may occur, it is possible to introduce extra error in the final calculated distance.

Figure 1. The diffuse reflection
[light.physics.auth.gr/enc/reflection.html]



(a)

The beam's footprint in a distance of 100m



(b)

Therefore the reflection of the material is also a main parameter. It is defined as the ratio of the beam that reflected back to the incident beam. The refractive index has a wide range from 0.05 which corresponds to materials like asphalt, to 0.97 for material like steel. However the refractive index might surpass the 1 for mirrors. The color of the material surface also affects the result (Zámecníková, 2014 b). The light –color surfaces give more accurate results as more beam percentage was reflected. On the contrary dark surfaces absorb a large quantity of the beam and gives poor results (Jetkins F., White H., 1981). Some others physical properties of materials as the electrical conductivity, the magnetic permeability and the roughness of their surface are influence the measurement. Due to these characteristics sometimes the beam is absorbed and is reflected from the inside body of the material and not from its surface.

The footprint of the laser beam must have a concrete shape like circle, ellipse or trapezoid. The deformation of the laser beam's shape due to its skew direction leads to gross errors in the RL distance measurement. Moreover as the distance is lengthened, the footprint of the laser becomes bigger and the accuracy of measurement is diminished as the wave of the radiation deformed and scatters in the space (Key & Lemmens, 2005). This deformation may cause an error to the measurement or cancel the measurement. A typical divergence angle of the emitted beam (average for the modern total station) of 5 arcminutes produces a beam footprint of 7cm radius in a distance of 100m (figure1b) (Junyu M et al, 2009). Also if the beam's footprint is larger than the size of target, for long distances, then there is the possibility to receive parts of beam from other surfaces laying ahead or back of the target.

So the size of the distance plays significant role as for short distances the results agreed with the manufacturer nominal accuracy (Lambrou et al., 2010). As the distance became longer errors are maximized. It is proved that for outdoors distances longer than 200m and incident angle larger than 30° there is no measurement or the measurement has large deviation (Khalil, R. 2015)

Last but not least is the parameter of the incident angle. The incident angle is defined as the deviation of the beam's direction from the perpendicular position to the material surface. When the beam incidents perpendicular to the surface its footprint is minimized and the maximum radiation returns to the total station with less multi – paths. If the incident angle skews due to the relative position between the instrument and the target or because of the target position (as external corner on a wall etc.), then the laser footprint is magnified and the reflected rays came from a larger area. So, the desired point is not clearly defined and the bias of the measurement is increased (Coaker, 2009, Schulz, 2007).

It is proved (Zámečníková, M. et al, 2014 a) that the incident angle is a significant parameter, which plays major role to the reliability of the RL distance measurement. However for indoors short distances up to 50m the results seem to be successful for about 80% of the measurements even for incident angle 45° (Lambrou et al, 2010). Actually the technology of the EDM's manufacture incorporates advanced systems in the new total stations in order to improve the RL measurements. It is worth to note that there are the well-known "time of flight" (Paiva, 2005) and "phase shift" methods (Trimble, 2005) and also the "system analyzer" method, purported to improve the accuracy and the credibility of RL measurements (Bayoud, 2006).

So, this work investigates the possibility of modeling the RL measurement deviations. The main idea is to acquire the differences between a prism measured distance and the corresponding RL ones in different incident angles in order to adopt equations to these differences for the correction of any measured RL distance.

Namely it aims to provide error equations **due to the incident angle of the beam, for outdoors measurements at 100m**. The process uses experimental data. For the experiment several materials are used as targets as well as three advanced total stations in order to acquire reliable results. Moreover the resulted equations are assessed by statistical indexes in order to be reliable.

2. Investigation Methodology

Twenty three materials with smooth and rough surfaces, of different colors were participating to the experiment. More specifically Kodak Gray Card, Kodak White Card, cement grey, cement white, paperboard black, paperboard yellow, plastic red, plastic white, tile grey, tile beige, tile brown, foam brown, foam white, marble, wood, melamine, asphalt, aluminium, aluminium blue, aluminium white, aluminium yellow, rock and chipboard. The material targets are put on a special supporting base (figure 2), which is adapted on the same adaptor as the prism. The special supporting base is manufactured for this kind of experiments and it is metrological tested for its reliability (Lambrou et al., 2010). The surface of each material belongs to the same vertical plan as the prism center when they are put on the special supporting base. So, the base is put on a leveled tribrach, it has rotation possibility and also it ensures vertical placement of the materials and identification of the prism's center with the materials surface.

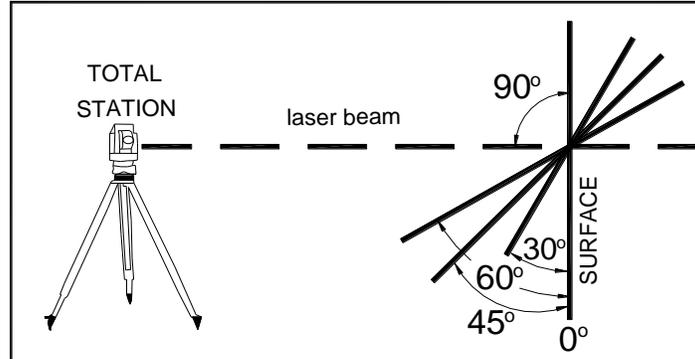
Figure 2. The special supporting base with the goniometer under it and the yellow paperboard on the special supporting base



The distance is measured firstly on the prism and continually the same distance is measured consequently on all the materials by changing their surface position towards the total station's sighting line per 5° each time.

Namely 13 incident angles of 0° , 5° , 10° , 15° , 20° , 25° , 30° , 35° , 40° , 45° , 50° , 55° and 60° are tested (figure 3). This number of measurements provides adequate observation equations in order to reach reliable approximation functions. Thus there are 23 different materials and 13 incident angles, a total of 299 measurements for each instrument (Dagianis, 2017). As incident angle 0° is considered the perpendicular position of the material to the sighting line of the instrument.

Figure 3. Different incidence angles of the laser beam on the material's surface



Each measurement is taken five times and the mean value is calculated in order to avoid gross errors or inadvertencies during the measurements.

The size of the distance is selected at 100m as it is a usual, mean and "safe" distance for RL measurements for the majority of the survey works. The experiment is implemented outdoors in order to be representative for the real measurement environmental conditions. The atmospheric conditions (temperature, pressure and humidity) are measured and the appropriate values are inserted in total station's processor.

The total stations that are used as well as their nominal accuracies σ_R for distance measurement on a prism are the Leica TCRM 1201⁺ with $\pm 1\text{mm} \pm 1.5\text{ppm}$, Leica TM30 with $\pm 1\text{mm} \pm 1\text{ppm}$ and Trimble VX with $\pm 2\text{mm} \pm 2\text{ppm}$. Their nominal accuracy σ_{RL} for the RL distance measurement is $\pm 2\text{mm} \pm 2\text{ppm}$. (Specifications for Leica TPS1200, TM30 and Trimble VX)

The differences $\Delta D_{i_{\text{meas}}}$ between the right distance D and the RL distances, on all the tested materials and incident angles are calculated as

$$\Delta D_{i_{\text{meas}}} = D - D_{RL_i} \quad (1)$$

Where D is the right distance with reflector

D_{RL_i} is the RL distance on every material and incident angle

For the data assessment, the expected and acceptable error for this difference is

$$\sigma_{\Delta D_{i_{\text{meas}}}} = \sqrt{\sigma_R^2 + \sigma_{RL}^2} \quad (2)$$

Where σ_R and σ_{RL} are the nominal errors for reflector and reflectorless distance measurement correspondingly. So, the following equation should be valid for confidence level 95% where $z_{95\%}$ is 1.96.

$$-z_{95\%} \cdot \sigma_{\Delta D_{i_{\text{meas}}}} \leq \Delta D_{i_{\text{meas}}} \leq z_{95\%} \cdot \sigma_{\Delta D_{i_{\text{meas}}}} \quad (3)$$

If $\Delta D_{i_{\text{meas}}}$ is included in this interval, so the measurement is correct. In any other case the measurement needs correction.

So, polynomial functions as the following are searched in order to find the best fitting to the acquired data.

- First order $y_i = a \cdot x_i + b$
- Second order (parabolic) $y_i = a \cdot x_i^2 + b \cdot x_i + c$ increasing $a > 0$ or decreasing $a < 0$
- Exponential $y_i = a \cdot e^{b \cdot x_i}$ or $y_i = -a \cdot e^{b \cdot x_i}$

Where x is the value of the incident angle in dec and y_i is the $\Delta D_{i_{\text{meas}}}$ in mm.

The following statistical criteria are used for the evaluation of the optimum correction function:

- ✓ The RMSE, which should take values less than the $\sigma_{\Delta D_{i_{meas}}}$ in order to be within the noise of the measurements.
- ✓ The R^2 correlation coefficient, which is the main criterion for the quality of the equation. If R^2 is close to 1, it indicates good adaptation quality.

According to the equation 3 and taking in to consideration the nominal accuracies of the involved total stations both with reflector and RL measurements, the acceptable limits are $\sigma_{\Delta D_{i_{meas}}} = \pm 4.4\text{mm}$ for Leica and $\sigma_{\Delta D_{i_{meas}}} = \pm 5.5\text{mm}$ for Trimble total station (for confidence level 95%).

3. Data processing

Table 1 presents the min and the max deviations $\Delta D_{i_{meas}}$ from the prism measurement for every material for 13 incident angles that each total station achieves. Also the number of the accepted measurements for each material as well as the total percentage of the accepted measurements for each total station is registered. The table indicates that differences up to 5cm are detected.

Table 1. Min and max $\Delta D_{i_{meas}}$ and accepted measurements for every total station for 23 materials and 13 incident angles

	Leica TCRM1201 ⁺			Leica TM30			Trimble VX			
	Min (mm)	Max (mm)	Accepted Out of 13	Min (mm)	Max (mm)	Accepted Out of 13	Min (mm)	Max (mm)	Accepted Out of 13	
Kodak Gray Card	2	38	3	2	7	11	5	19	1	
Kodak White Card	2	34	4	0	6	11	5	18	2	
Cement white	3	28	2	0	4	13	8	16	0	
Cement grey	2	32	3	1	8	11	8	14	0	
Paperboard yellow	3	38	2	1	7	11	3	7	8	
Paperboard black	4	8*	1	0	12	11	3	8	8	
Plastic white	0	38	4	1	20	3	5	13	1	
Plastic red	1	40	5	0	14	2	7	12	0	
Marble	0	43	4	0	8	7	12	20	0	
Tile beige	1	41	3	0	11	5	8	13	0	
Tile grey	1	40	3	0	10	5	4	9	5	
Foam white	1	37	5	1	7	2	11	14	0	
Foam brawn	2	38	2	0	1	13	5	11	2	
Wood	2	41	2	0	2	13	8	13	0	
Chipboard	2	42	2	0	4	13	4	13	2	
Melamine	1	37	2	0	2	13	5	18	2	
Tile	1	42	3	0	2	13	11	22	0	
Asphalt	5	44	0	0	9	7	5	10	1	
Rock	0	43	3	0	2	13	9	15	0	
Aluminium	1	43	3	0	8	10	5	30	1	
Aluminium white	1	40	3	0	2	13	9	26	0	
Aluminium blue	1	47	2	2	9	2	18	29	0	
Aluminium yellow	3	22	4	0	19	9	8	22	0	
<i>Accepted percentage for each TC</i>			22%				70%			11%

Table 2 presents the min and max deviations $\Delta D_{i_{meas}}$ that each incident angle provides for all the tested materials and the number and the total percentage of accepted measurements for each incident angle for the three instruments. For incident angle up to 10° about the half of measurement are within the permissible error.

For incident angle 20° only the one third of measurements are acceptable while for incident angle 60° this percentage declines to only 13%. Thus it is obvious the significance of the incident angle in RL measurements.

Moreover figure 4 illustrates the percentage of not acceptable measurements for every incident angle for the three total stations and figures 5, 6 and 7 displays the deviations $\Delta D_{i_{meas}}$ that each total station provides for all the materials and incident angles.

Table 2. The min and the max $\Delta D_{i_{meas}}$ and the accepted measurements for every incident angle for 23 materials

dec	Total Accepted	Leica TCRM1201 ⁺			Leica TM30			Trimble VX		
		Min mm	Max mm	Accepted Out of 23	Min mm	Max mm	Accepted Out of 23	Min mm	Max mm	Accepted Out of 23
0	75%	0	8	20	0	6	22	3	18	10
5	70%	0	7	19	0	7	22	3	10	7
10	55%	1	9	15	0	6	20	3	19	3
15	42%	1	10	6	0	7	19	3	21	4
20	36%	0	12	3	0	7	19	4	22	3
25	29%	2	13	1	0	8	17	4	23	2
30	29%	3	16	1	0	8	17	5	26	2
35	25%	6	17	0	0	8	15	5	27	2
40	20%	10	32	0	0	11	14	6	29	0
45	20%	12	38	0	0	14	14	6	30	0
50	19%	13	41	0	0	15	13	7	29	0
55	13%	16	41	0	0	17	9	7	30	0
60	13%	22	47	0	0	20	9	7	27	0

Figure 4. Not acceptable measurements for every incident angle

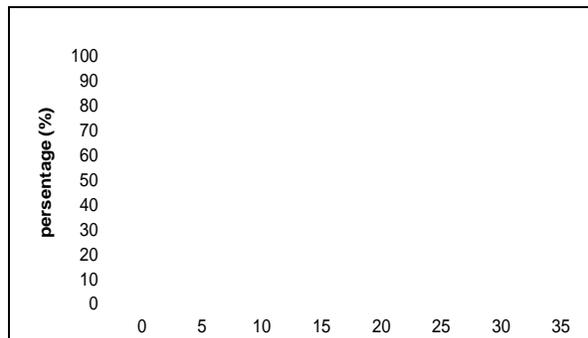


Figure 6. The differences $\Delta D_{i_{meas}}$ of TM30

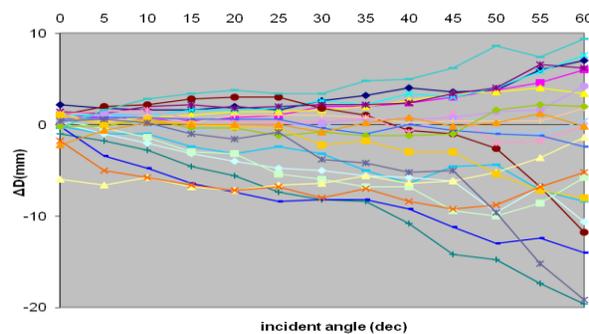


Figure 5. The differences $\Delta D_{i_{meas}}$ of TCRM1201⁺

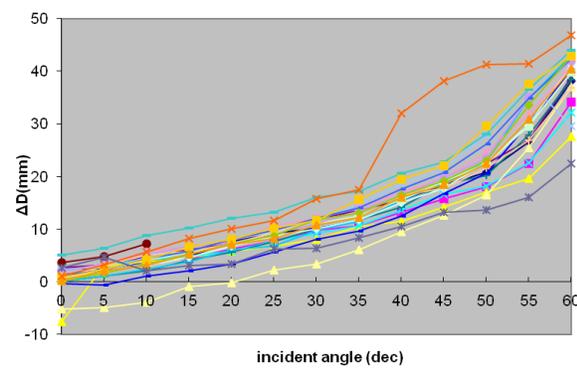
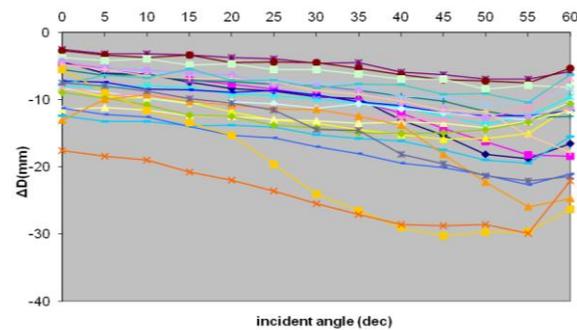


Figure 7. The differences $\Delta D_{i_{meas}}$ of Trimble VX



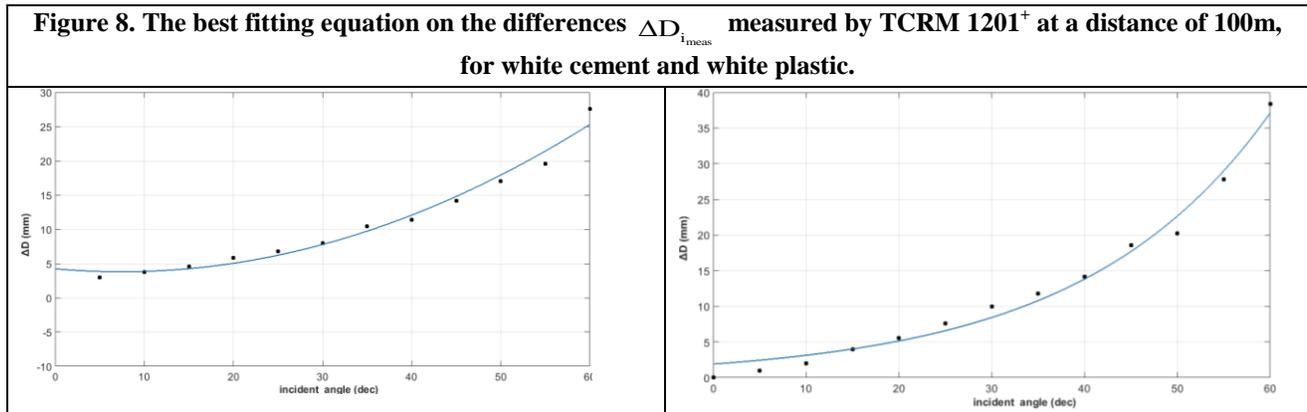
Thus for each total station and for each material that the measured distance exceeds the permissible error the best fitting correction equation is adapted to the differences $\Delta D_{i_{meas}}$. Moreover for each total station the goal is to group the equations which come out in order to be easier the correction procedure.

For Leica TCRM 1201⁺ almost all the materials give acceptable results up to 10° but beyond that all they need correction equations. The 22% of measured distances are within the permissible interval. It is underlined that the black paperboard fails to measure for incident angle greater than 10°.

So, 22 equations are calculated. In all cases TCRM 1201⁺ measures shorter distances on reflectorless mode. Table 3 presents the results of these calculations as figure 8 illustrates the adaptation equation for white cement and white plastic.

Table 3. The correction equation for each material for TCRM 1201⁺

materials	function	p1	p2	p3	p4	R ²	RMSE (mm)
Kodak Gray	f(x)=p ₁ · e ^{p₂·x}	2.5	0.0443	-	-	0.9841	±1.4
Kodak white		2.2	0.0442	-	-	0.9750	±1.5
cement white	f(x)=p ₁ · x ² +p ₂ · x+p ₃	0.0078	-0.1150	4.2	-	0.9795	±1.2
cement grey		0.0083	-0.0741	3.4	-	0.9743	±1.6
Paperboard yellow	f(x)=p ₁ · e ^{p₂·x}	3.0	0.0417	-	-	0.9852	±1.3
plastic white		1.9	0.0494	-	-	0.9860	±1.4
plastic red		1.1	0.0602	-	-	0.9929	±1.1
marble		1.7	0.0540	-	-	0.9920	±1.2
Tile beige		2.1	0.0494	-	-	0.9916	±1.1
tile grey		2.5	0.0452	-	-	0.9907	±1.1
foam white	f(x)=p ₁ · x ² +p ₂ · x+p ₃	0.6	0.0688	-6.6	-0.0518	0.9961	±0.9
Foam brawn	f(x)=p ₁ · e ^{p₂·x}	2.9	0.0463	-	-	0.9916	±1.1
wood		3.2	0.0420	-	-	0.9924	±1.1
chipboard		2.7	0.0456	-	-	0.9898	±1.3
melamine		3.0	0.0410	-	-	0.9835	±1.4
tile		2.9	0.0448	-	-	0.9949	±1.0
asphalt		5.4	0.0344	-	-	0.9876	±1.4
rock		2.3	0.0484	-	-	0.9873	±1.5
aluminium		3.2	0.0438	-	-	0.9912	±1.3
aluminium white		2.3	0.0477	-	-	0.9904	±1.2
aluminium blue		f(x)=p ₁ · x ³ +p ₂ · x ² +p ₃ · x+p ₄	-0.0003	0.0344	-0.2546	3.0	0.9703
Aluminium yellow	f(x)=p ₁ · e ^{p₂·x}	2.1	0.0385	-	-	0.9685	±1.1



TM30 has exceptional measurements for the majority of the combinations. The 70% of the measured distances are within the permissible error. So, for only 13 materials correction equations are needed. For the differences, $\Delta D_{i_{meas}}$ which are out of the limit the reflectorless distance is longer.

Table 4 presents the results of these calculations as figure 9 illustrates the adaptation equations for yellow aluminum and yellow paperboard. Finally VX has the worst results as there are only 11% of acceptable distances and all the materials need correction equations. In all cases it measures longer distances on reflectorless mode.

Table 5 presents the results of these calculations as figure 10 illustrates the adaptation equations for wood and grey cement. The adapted equations for all the materials are statistically accepted as provide RMSEs less than ± 2 mm, which is equal to the RL nominal accuracy of the three instruments and less of the $\sigma_{\Delta D_{i_{meas}}}$. Moreover almost all the R² are greater of 0.9, which means that the approximations are reliable.

Table 4. The correction equations for each material for TM30

materials	function	p1	p2	p3	p4	R ²	RMSE(mm)
cement grey	$f(x)=p_1 \cdot x^2+p_2 \cdot x+p_3$	0.0023	-0.0429	1.3	-	0.9382	±0.5
Paperboard yellow	$f(x)=p_1 \cdot e^{p_2 \cdot x} + p_3 \cdot e^{p_4 \cdot x}$	1.4	-0.0068	0.2	0.0578	0.9184	±0.6
Paperboard black	$f(x)=p_1 \cdot x^2+p_2 \cdot x+p_3$	-0.0080	0.3047	0.4	-	0.9629	±0.9
plastic white		-0.0024	-0.1597	-1.1	-	0.9904	±0.6
plastic red	$f(x)=p_1 \cdot x+p_2$	-0.1960	-2.3	-	-	0.9360	±1.0
marble	$f(x)=p_1 \cdot x^2+p_2 \cdot x+p_3$	-0.0004	-0.0984	-0.4	-	0.8767	±1.0
Tile beige	$f(x)=p_1 \cdot x+p_2$	-0.1512	-0.4	-	-	0.9086	±1.0
tile grey	$f(x)=p_1 \cdot x^2+p_2 \cdot x+p_3$	0.0033	-0.3509	1.3	-	0.8954	±1.2
foam white		0.0030	-0.1282	-5.6	-	0.8450	±0.7
asphalt	$f(x)=p_1 \cdot x+p_2$	0.1354	0.5	-	-	0.9082	±0.9
aluminium	$f(x)=p_1 \cdot x^2+p_2 \cdot x+p_3$	-0.0027	0.0207	0.5	-	0.9706	±0.6
aluminium blue		0.0047	-0.3394	-2.5	-	0.8694	±0.8
Aluminium yellow		-0.0075	0.1678	-0.6	-	0.9482	±1.5

Figure 9. The best fitting equation on the differences $\Delta D_{i_{meas}}$ measured by TM 30 at a distance of 100m, for yellow aluminum and yellow paperboard

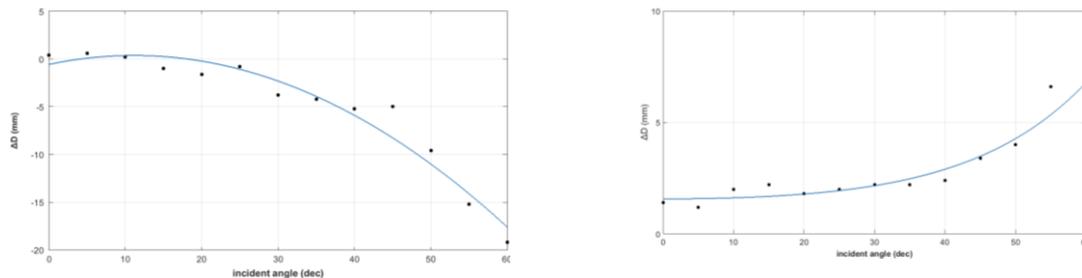
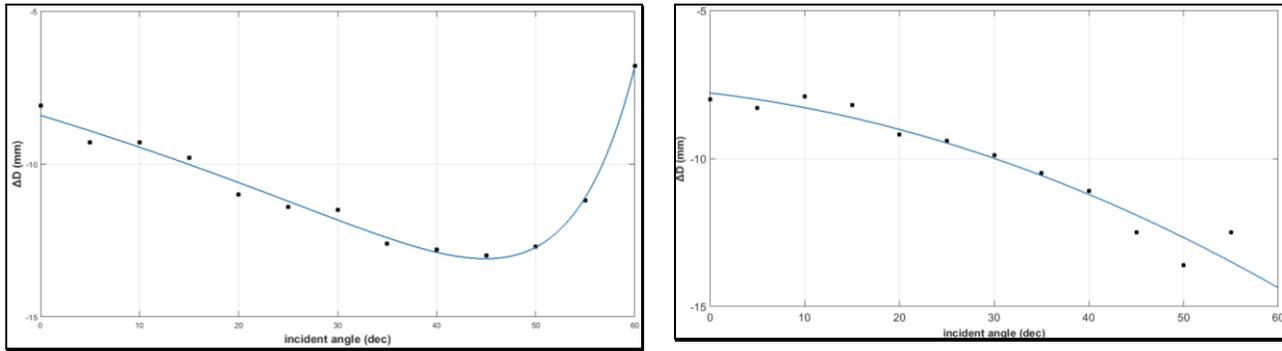


Table 5. The correction equation for each material for VX

materials	function	p1	p2	p3	p4	R ²	RMSE(mm)
Kodak Gray	$f(x)=p_1 \cdot x^2+p_2 \cdot x+p_3$	-0.0016	-0.1453	-4.6	-	0.9314	±1.4
Kodak white		-0.0027	-0.1	-4.9	-	0.9884	±0.6
cement white	$f(x)=p_1 \cdot e^{p_2 \cdot x} + p_3 \cdot e^{p_4 \cdot x}$	0.01	0.1269	-8.5	0.0164	0.9898	±0.3
cement grey	$f(x)=p_1 \cdot x^2+p_2 \cdot x+p_3$	-0.0012	0.0	-7.8	-	0.9360	±0.5
Paperboard yellow	$f(x)=p_1 \cdot x+p_2$	-0.0848	-2.3	-	-	0.9421	±0.4
Paperboard black		-0.0892	-2.5	-	-	0.9464	±0.4
plastic white	$f(x)=p_1 \cdot x^2+p_2 \cdot x+p_3$	-0.0012	0.0	-5.8	-	0.9795	±0.4
plastic red	$f(x)=p_1 \cdot x+p_2$	-0.0921	-7.1	-	-	0.9317	±0.5
marble	$f(x)=p_1 \cdot x^2+p_2 \cdot x+p_3$	-0.0019	-0.0185	-12.8	-	0.9850	±0.3
Tile beige	$f(x)=p_1 \cdot x+p_2$	-0.0729	-8.8	-	-	0.9169	±0.4
tile grey		-0.0821	-3.5	-	-	0.9477	±0.4
foam white	$f(x)=p_1 \cdot e^{p_2 \cdot x} + p_3 \cdot e^{p_4 \cdot x}$	0.0	0.1235	-11.2	0.0055	0.9349	±0.3
foam brawn	$f(x)=p_1 \cdot x^2+p_2 \cdot x+p_3$	-0.001	-0.0725	-4.9	-	0.9640	±0.4
wood	$f(x)=p_1 \cdot e^{p_2 \cdot x} + p_3 \cdot e^{p_4 \cdot x}$	0.002	0.1441	-8.4	0.0118	0.9851	±0.3
chipboard	$f(x)=p_1 \cdot x+p_2$	-0.1458	-4.1	-	-	0.9357	±0.8
melamine	$f(x)=p_1 \cdot x^2+p_2 \cdot x+p_3$	-0.0033	0.0067	-5.5	-	0.9717	±0.7
tile	$f(x)=p_1 \cdot x+p_2$	-0.1908	-11.2	-	-	0.9723	±0.7
asphalt	$f(x)=p_1 \cdot x^3+p_2 \cdot x^2+p_3 \cdot x+p_4$	-1.5	-11.8100	-6.2	0.0062	0.4000	±1.3
rock	$f(x)=p_1 \cdot x^2+p_2 \cdot x+p_3$	0.0050	-0.3600	-8.14	-	0.9121	±0.7
aluminium		0.0079	-0.8984	-3.7	-	0.9585	±2.0
aluminium white		-0.0086	0.2592	-11.9	-	0.9449	±1.5
aluminium blue	$f(x)=p_1 \cdot x+p_2$	-0.2455	-17.4	-	-	0.9689	±0.8
aluminium yellow	$f(x)=p_1 \cdot e^{p_2 \cdot x} + p_3 \cdot e^{p_4 \cdot x}$	0.8	0.0914	-6.7	0.0292	0.9115	±1.4

Figure 10. The best fitting equation on the differences $\Delta D_{i_{meas}}$ measured by VX at a distance of 100m for wood and grey cement



The frequency of the occurrence of each kind of equation for each total station is illustrated in figure 11. For TCRM1201⁺ more equations are exponentials as the 2nd order is the majority for TM30 and VX. Figure 12 presents the total occurrence frequency of each order of equation.

Figure 11. The occurrence frequency of each equation for each total station

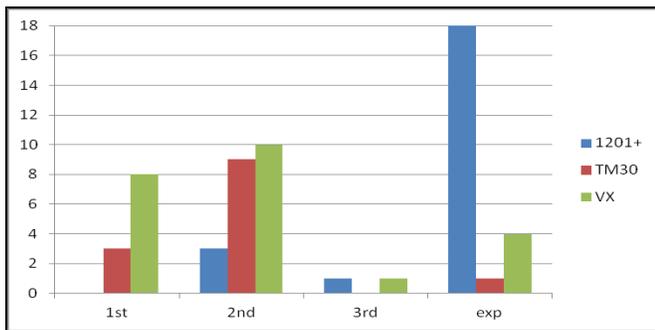
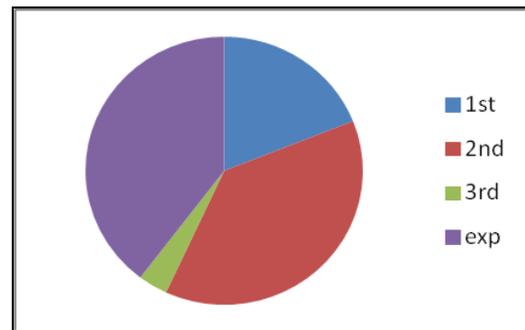


Figure 12. The total occurrence frequency of each equation



As it is mentioned above the idea is to group these approximation equations in order to be feasible a consolidated correction for each instrument measurements. Thus it is attempted to find one equation which approximates well the differences $\Delta D_{i_{meas}}$ for more than one material. The main criterion for this procedure is that the RMSE of the total equation should be less than the $\sigma_{\Delta D_{i_{meas}}}$ and also R^2 should have a statistical satisfactory value.

For TCRM 1201⁺ two groups were created the first one of 18 materials as the second of 3 materials. The first group includes the following materials: Kodak Gray, Kodak white, paperboard yellow, plastic white, plastic red, marble, tile beige, tile grey, foam brawn, wood, chipboard, melamine, tile, asphalt, rock, aluminium, aluminium white and aluminium yellow. So, equation 4 is come out.

$$f(x)=p_1 \cdot e^{p_2 \cdot x} = 2.6 \cdot e^{0.4473 \cdot x} \tag{4}$$

with $R^2=0.9318$ and $RMSE=\pm 3mm$

The second group includes only three materials, cement white, cement grey and foam white. The equation 5 is resulted.

$$f(x)=p_1 \cdot x^2+p_2 \cdot x+p_3 = 0.07923 \cdot x^2+0.01108 \cdot x-0.7 \tag{5}$$

with $R^2=0.8887$ and $RMSE=\pm 3.5mm$

Although TM 30 has the best result, where only 13 materials need corrections, its behavior is not systematic. Thus some materials are needed positive corrections and some others are needed negative ones. So only nine materials out of 13 it was feasible to be grouped.

These are paperboard black, plastic white, plastic red, marble, tile beige, tile grey, foam white, aluminium and aluminium blue. Equation 6 describes the result.

$$f(x)=p_1 \cdot x^2+p_2 \cdot x+p_3 = -0.0001612 \cdot x^2-0.1291 \cdot x - 1.05 \tag{6}$$

With $R^2=0.8719$ and $RMSE=\pm 3.4 mm$

For VX, which has more systematic deviations, it was feasible that twenty one materials, almost all that were tested, to be grouped. These materials are: Kodak Gray, Kodak white, cement grey, cement white, paperboard yellow, plastic white, plastic red, marble, tile beige, tile grey, foam white, foam brawn, wood, chipboard, melamine, tile, rock, aluminium, aluminium white, aluminium blue and aluminium yellow. The following equation gives the appropriate corrections.

$$f(x)=p_1 \cdot x^2+p_2 \cdot x+p_3 = 0.0006836 \cdot x^2 - 0.1868 \cdot x - 7.268 \tag{7}$$

with $R^2=0.8809$ and $RMSE=\pm 4mm$

4. Determination of a total equation for each total station

However, as the above material selection for each adaptation may be considered as arbitrary another approximation could be the following.

For each incident angle the mean value ΔD_{m_i} of the differences $\Delta D_{i_{meas}}$ (i=1 to 23) is calculated by using all the materials. So, there are 13 such ΔD_{m_i} to which an equation could be adapted in order to conclude to one mean correction equation.

For TCRM 1201⁺ the second order mean equation 8 is resulted

$$f(x)=p_1 \cdot x^2+p_2 \cdot x+p_3 = 0.09976 \cdot x^2 - 0.04526 \cdot x - 2.465 \tag{8}$$

with $R^2=0.9853$ and $RMSE=\pm 1.5mm$

For TM 30 due to its irregular behavior only the materials that needs corrections are used namely the materials which were presented corrections beyond the $\sigma_{\Delta D_{i_{meas}}}$. Thus the following mean equation is resulted

$$f(x)=p_1 \cdot x^2+p_2 \cdot x+p_3 = -0.0001099 \cdot x^2 - 0.132 \cdot x - 1.03 \tag{9}$$

with $R^2=0.9923$ and $RMSE=\pm 0.3mm$

Also for VX by using all the materials the second order mean equation 10 is resulted

$$f(x)=p_1 \cdot x^2+p_2 \cdot x+p_3 = -0.0006753 \cdot x^2 - 0.1788 \cdot x - 7.004 \tag{10}$$

with $R^2=0.9327$ and $RMSE=\pm 0.8mm$

Figure 13 illustrates the adaptation equations for TCRM 1201⁺ for 18 materials (a) and for the mean ΔD_{m_i} (b), TM 30 for 9 materials (c) and for the mean ΔD_{m_i} (d) and VX for 21 materials (e) and for the mean ΔD_{m_i} (f).

In order to evaluate the above equations 4, 6,7,8,9 and 10, the corrections $\Delta D_{i_{calcul}}$ that they give for each incident angle are calculated.

So the residuals u_i are registered as the difference between $\Delta D_{i_{meas}}$ and $\Delta D_{i_{calcul}}$ (equation 11).

Thus the index σ_0 is calculated for the above six equations 4, 6,7,8,9 and 10 (table 6).

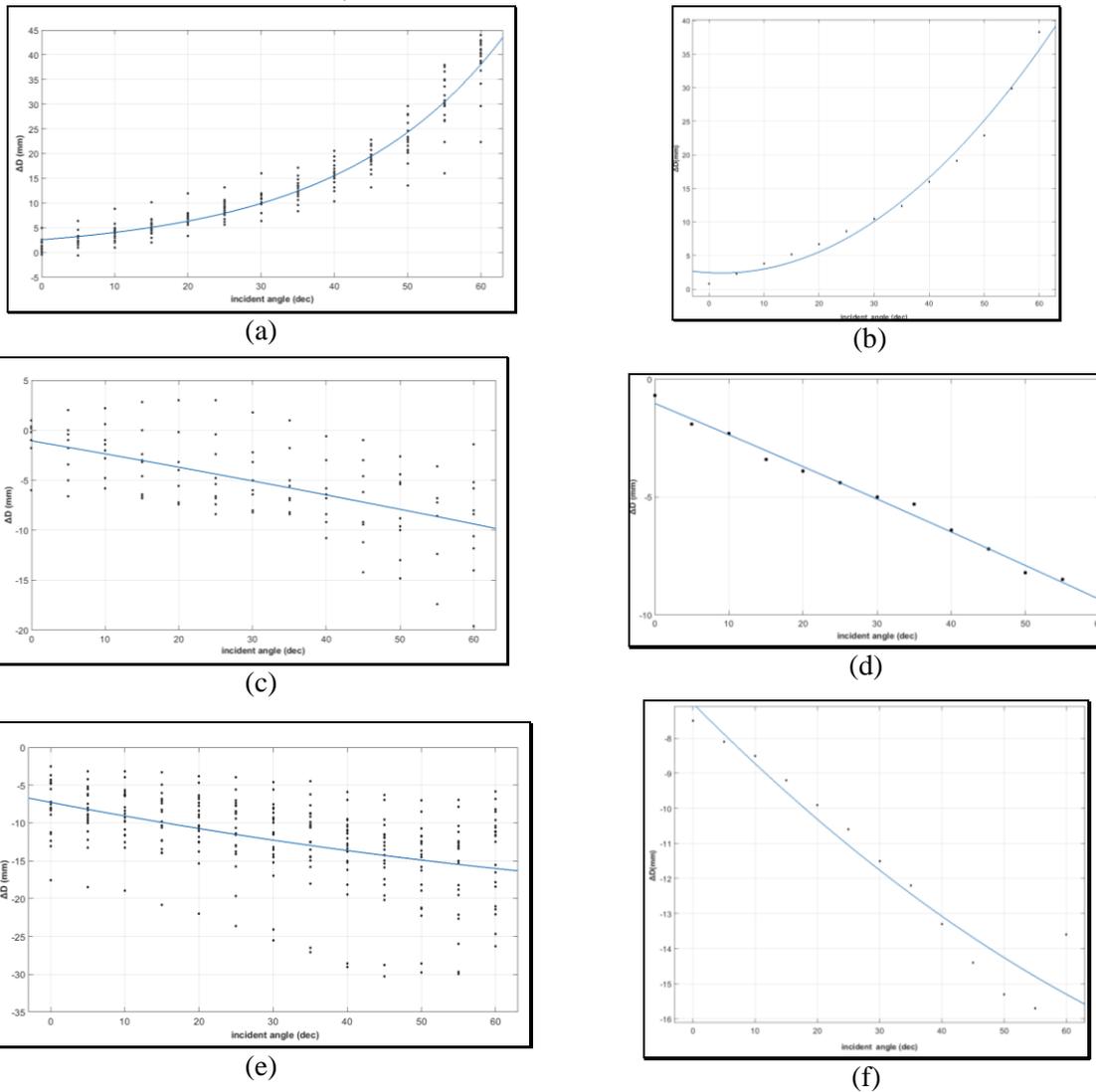
$$u_i = \Delta D_{i_{calcul}} - \Delta D_{i_{meas}}, \quad \sigma_0 = \pm \sqrt{\frac{\sum u_i u_i}{n-1}} \tag{11}$$

Table 6. The σ_0 of the adapted equations for some selected and for all the materials

	Number of materials		all materials
TCRM 1201+	18	$\sigma_0=\pm 4.3mm$	$\sigma_0=\pm 4mm$
TM30	9	$\sigma_0=\pm 3mm$	$\sigma_0=\pm 3mm$
VX	21	$\sigma_0=\pm 5.4mm$	$\sigma_0=\pm 5.2mm$

The results of the table 6 indicate that the corrections that came out for both procedures provide almost the same correction accuracy which is within the threshold of $\sigma_{\Delta D_{i_{meas}}}$. Consequently the RL distance measurement error due to the incident angle change can be expressed by a second order equation, as the three final mean equations are calculated.

Figure 13. The adaptation equations for TCRM 1201⁺ for 18 materials (a) and for the mean ΔD_{m_i} (b), TM 30 for 9 materials (c) and for the mean ΔD_{m_i} (d), VX for 21 materials (e) and for the mean ΔD_{m_i} (f)



5. Conclusions

This work managed to find an equation for the correction of the measured distances by using the reflectorless mode due to the incident angle influence. Three advanced total stations are tested outdoors at a distance of 100m with 23 materials in 13 different incident angles. This provides the possibility to make numerous of approximations with adequate freedom degree in order to end up to reliable results. The 23 materials are selected as the more usual meet in the surveys field work environment. Also the different colors for the same material give the average influence of this parameter to the approximations. Several equations of first, second, third order and exponential are used in order to approximate better the deviation from the correct distance measurement as the incident angle of the laser beam to the surface becomes greater. Differences up to 5cm are registered between the reflector and reflectorless mode. For incident angle of the laser beam less than 10° these deviations are about 1cm. The experiments show that on average a small percentage about 20% of the RL measurements for several incident angles are within the nominal accuracy of the manufacturer. Also for perpendicular sightings the 75% of the measurements are correct but this declines to 55% as the incident angle becomes 10° and to 13% when the incident angle becomes 60°.

Each total station presents different behavior on RL mode in the distance of 100m as TCRM 1201⁺ measures always shorter distances, VX measures always longer distances and TM30 measures shorter distances when the difference is within the acceptable error $\sigma_{\Delta D_{i_{meas}}}$ while it measures longer distances when the difference exceeds the $\sigma_{\Delta D_{i_{meas}}}$

TM30 proved to be an especially reliable total station as has in total 70% acceptable RL measurements and also its greater deviation is 2cm for all the materials and all the tested incident angles. The approximation equation that was calculated for every single material can correct the measured distance accurately by ± 1 mm. The 2nd order equations approximate well the more materials' deviations separately as well as the mean differences of all the materials in each incident angle for each instrument. The use of the mean deviation ΔD_{m_i} for each incident angle which is calculated from all the materials leads to a reliable approximation equation which can correct any RL measurement. This total equation that is determined for each total station can correct its RL mode measurements by satisfying accuracy of ± 3 mm to ± 5 mm. This study proves that it is feasible to model and to correct the RL distance measurements due to the incident angle influence up to some mm. So, this feature of the modern total stations can be used for more accurate measurements as technical constructions monitoring or stake outs where the accuracy requirements are greater.

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