# Mathematical Modeling and Software Development On Pertubation Effects Towards Small Satellite Mission On Low Earth Orbit

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## Abstract

*The orbit of satellite cannot be calculated using the theoretical method for precise calculation. This is because the orbital elements will deviate from true elements with time. Regression of the lines-of-the-nodes and rotation of the line-of-apsides (major axis) are the two principal effect of the perturbation of low altitude orbits due to the oblate shape of the earth. This gravitational effect of an oblate earth can be modeled by using Spherical Harmonics to satisfy the Laplace's Equation. A precise calculation of the perturbations is possible only if the orbit is sufficiently well known. The results can be obtained by using Ordinary Differential Equation, i.e., Runge-Kutta method, and Matlab to view the graphs. This paper discusses the development of Mathematical Modeling and Analysis Software on Perturbation Effects Using Cowell's Method and with the help of Matlab to visualize the Effect.*

**Keywords***:* small satellite, pertubation effects, mathematical model, low earth orbit

## *Introduction*

A perturbation is a deviation from some normal or expected motion. In case of a satellite, the actual path will vary from the theoretical two-body path due to perturbations caused by other mass bodies (such as Moon) and additional forces not considered in Keplerian Motion, such as the oblateness potential of the earth. One cannot regard that the perturbations are always small, for they can be as large as or larger than the primary attracting forces. In fact, many interplanetary missions would miss their target entirely if the perturbing effects were not taken into account (Baker, 1967). The objective of this paper is to present the well-known perturbing technique, which is the method of Cowell. In addition, numerical integration and simulation is done using the Graphical User Interface (GUI) of Matlab.

## *The Method of Cowell*

The method of Cowell is the most widely practiced today of all the numerical methods of integrating the differential equations of perturbed motion. The method of Cowell is taken and derived from the Gauss equation (Bate,R.R, et. all, 1971). The application of Cowell's method is simply to write the equations of motion of object (of a two bodies problem), with the perturbations, and then to integrate them step-by-step numerically. For a two body problems without perturbations, the equation is

$$\bar{r}'' + \frac{\mu}{r^3}\bar{r} = 0 \tag{1}$$

For a two body problems with perturbations, Equation (1) will be added with the perturbing acceleration as in Equation (2)

$$\bar{r}'' + \frac{\mu}{r^3}\bar{r} = a_p \tag{2}$$

This is a second order differential equation. For numerical integration purpose, Equation (2) would be reduced to first-order differential equations as below;

$$\bar{r}' = \bar{v}$$
$$\bar{v}' = \bar{a}_p - \frac{\mu}{r^3}\bar{r} \tag{3}$$

where $\bar{r}$ and $\bar{v}$ are the radius and velocity of a satellite with respect to the larger central body. In vector components, Equation (3) would look like as in Equation (4),

$$x' = v_x \qquad v'_x = a_{px} - \frac{\mu}{r^3} x$$

$$y' = v_y \qquad v'_y = a_{py} - \frac{\mu}{r^3} y$$

$$z' = v_z \qquad v'_z = a_{pz} - \frac{\mu}{r^3} z$$

(4)

where $r = (x^2 + y^2 + z^2)^{1/2}$. The perturbing acceleration, $\bar{a}_p$ could be due to the presence of other gravitational bodies such as the Moon, Sun and other planets, and gravitational effects such as the oblateness of the earth. If the Moon is regard as the perturbing body, the equations would be

$$x' = v_x \qquad v'_x = -\frac{\mu}{r^3} x - \mu_m \left( \frac{x_{MS}}{r_{MS}^3} - \frac{x_{ME}}{r_{mE}^3} \right)$$

$$y' = v_y \qquad v'_y = -\frac{\mu}{r^3} y - \mu_m \left( \frac{y_{MS}}{r_{MS}^3} - \frac{y_{ME}}{r_{mE}^3} \right)$$

$$z' = v_z \qquad v'_z = -\frac{\mu}{r^3} z - \mu_m \left( \frac{z_{MS}}{r_{MS}^3} - \frac{z_{ME}}{r_{mE}^3} \right)$$

(5)

where r is the radius from Earth to the satellite, *rMS* is the radius from Moon to satellite, *rME* is the radius from Moon to Earth and *μM* is the gravitational parameter of the Moon (Figure 1). The coordinate of the Moon can be found using the Lunar Almanac or fixing the Moon's location (M.S. Kamal, et. all, 1998). Equation (5) is the equation for the perturbation for third body perturbation, i.e., Sun and other planets. The indicator has to be changed according to the third body positions. The gravitational effect of an oblate shape of the Earth can more easily be visualized by picturing a spherical earth surrounded by a belt of excess matter representing the equatorial bulge. When a satellite is in the positions shown in Figure 2, the net effect of the bulges represents the oblate and unsymmetrical shape of the Earth. This will produce a small torque on the satellite about the center of the Earth. This torque will cause the plane of orbit to deviate from its original plane (Bate,R.R, et. all, 1971). The oblateness potential of the Earth can be modeled by using the Spherical Harmonics functions, *U*, as shown in Equation (6) (Kee C.D., 1997)

$$U(r, \lambda, \phi) = \frac{\mu}{r} \left[ 1 - J_2 \left( \frac{R}{r} \right)^2 \frac{(3\sin^2\phi - 1)}{2} - J_3 \left( \frac{R}{r} \right)^3 \frac{(5\sin^3\phi - 3\sin\phi)}{2} \dots \right]$$

(6)

where $R$ = 6,378 km (Earth semi-major axis), $\mu$ = 398,601 km3/s2 (GM of Earth), $J2$ = 1082.64 x 10-6, $J3$ = -2.56 x 10-6 and φ = geocentric latitude  The total perturbing accelerations then is the summations of the major influence of the third body  and the effects of oblateness of the Earth, such as J2 and J3
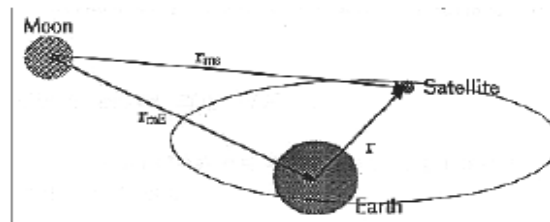


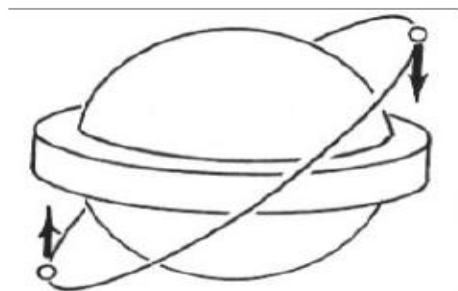Figure 1 : The relationships (positions) between Earth, Moon & satellite



Figure 2:  Perturbation example of perturbative torque caused by earth's equatorial bulge

### Result and Discussion

Any second order differential equation (ODE) can be solved with numerical integration methods. There are many methods to solve this ODE problems, such as, Method of Runge-Kutta, Gill, Euler-Cauchy, Bowie, and etc. However, the use of Runge-Kutta method is widely known as the most direct, stable and do not require a starting procedure. In Matlab, the use of Runge-Kutta method as one of the numerical integration methods has made the calculations easier to calculate (S. Nakamura, 1993). In Matlab programming, the use of this method can be seen by the function of ode45, which is based on an explicit Runge-Kutta (4,5) formula. In general, ode45 is the best solution to apply to the most of the differential equation. To solve the second order differential equations of perturbations, initial state, which are the positions and velocity have to be specified. Table (1) shows the initial state of a specified satellite. To view the simulation, a Matlab based software is developed for this purpose. Figure (4) until Figure (7) shows the result of perturbation effects of a specified satellite.

**Table 1:** Initial state of a specified satellite

| Initial Positions (km) | X | Y | Z |
|---|---|---|---|
| | 11335 | -7740 | 941 |

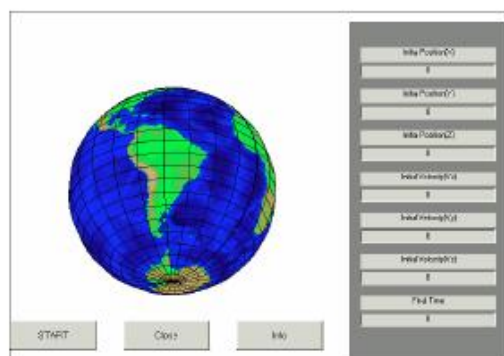| Initial Velocity (km/s) | Vx | Vy | Vz |
|---|---|---|---|
| | -0.9199 | 6.859 | -0.3798 |



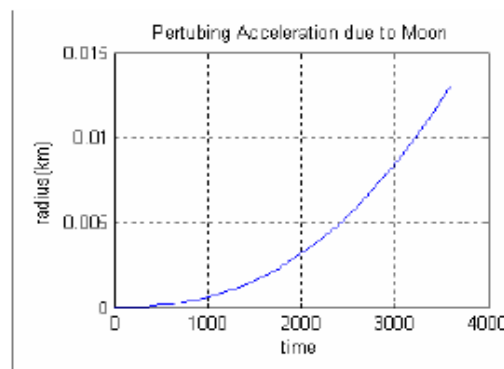Figure 3: The Perturbation Window – Input



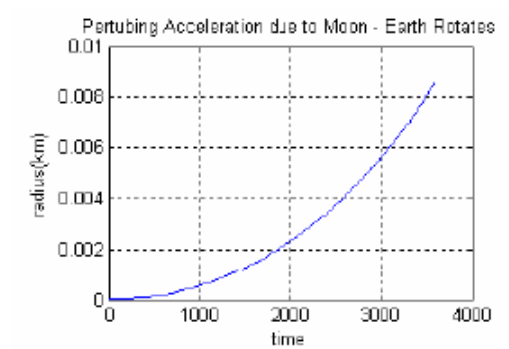Figure 4: The deviation of orbit due to the Moon Perturbing acceleration



Figure 5. The deviation of orbit due to the Moon Perturbing acceleration (Moon Rotates)
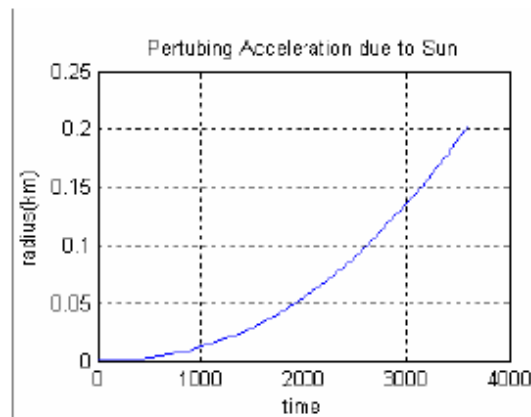
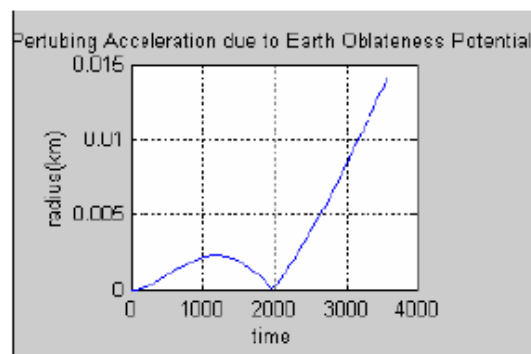Figure 6: The deviation of orbit due to the Sun's
perturbing acceleration.



Figure 7: Perturbing acceleration effect due to the
Earth Oblateness Potential

### Conclusion and Further Work

Cowell's method, which is the simplest and most straight forward in calculating perturbation, can be used to calculate the perturbed magnitude which the orbit of a satellite is perturbed although in a short time, such as an hour. The perturbed magnitudes obtained in this paper are almost large for one hour duration of orbit. Therefore, in order to have a precise calculation of a satellite orbit, perturbing accelerations cannot be neglected. Spherical Harmonics function is used to model the Earth Gravity Potential, in order to have the better equation used in calculating the perturbing acceleration. The usage of Matlab as the programming tools proved to be well-satisfactory because Matlab is a powerful programming tool for calculating the numerical integration and a userfriendly simulation tools through Graphical User Interface (GUI).

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