Lesson Study: Collaboration, Improvement, and Reflection

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Introduction

Teaching mathematics at a small liberal arts college provides unique challenges. Our mathematics department totals only four instructors---three of whom are relatively new to the campus---all teaching different courses. This creates the risk for our day-to-day activities to become isolated from the broader goals of the departmentand possibly leading to inconsistencies between consecutive courses to develop. Moreover, staying narrowly focused on our individual courses creates the potential for a less collegial work environment. In order to confront these challenges, our department conducted a lesson study as a vehicle to facilitate interdepartmental communication and to familiarize ourselves with the needs and attitudes of our students. Traditionally, lesson study is used for producing "canned" lessons that multiple instructors can perfect and share; for us, the project was also intended to provide an opportunity to collaborate. This article details the process used and the observations made from the study.

The focus of our lesson study was a single day's content from our core College Algebra course. This was the obvious choice, as most of our students must take it as part of the Associates of Arts degree requirement. Our institutional success rate was approximately 61% in Fall 2011---at the higher end of established national attrition rates of 40-60% (Adelman, 2004; Small, 2010). The lesson study was led by Sarah Bennett who was also the instructor for the course. The remaining co-authors participated as observers in the study.

Planning and Presentingthe Lesson

The course instructor spent roughly half an hour planning the lesson on her own. She prepared PowerPoint slides,which included content on each topic, examples for the students to work on in groups, and solutions of the examples for later discussion. The instructor planned to complement the slides throughout the lesson by further explaining the ideas verbally and making use of the whiteboard. The students would be expected to actively participate by completing exercises presented on the slides in groups of three or four. This would be followed by an instructor-led discussion of their solutions. The planned format of the lesson was standard practice for this class.

This lesson plan was presented to the group during an hour-long discussion. This was an important stage--not only in helping to improve the lesson--but also asa good opportunity for the members of our small department to discuss pedagogy. We had considerable discussion centered around the amount of content on the slide. This led to a discussion about the effective use of PowerPoint in the classroom. As a result, the instructor made the slides less dense in orderto improves comprehension. Further feedback from the planning team resulted in adding a summary of key concepts with the goal of helping the students reflect on what they had learned during the lesson. The lesson consisted of discussing the following algebraic topics:

- the graph of the square root function $y = \sqrt{x}$;
- reflections of graphs across lines;
- and piecewise-defined functions and their graphs.

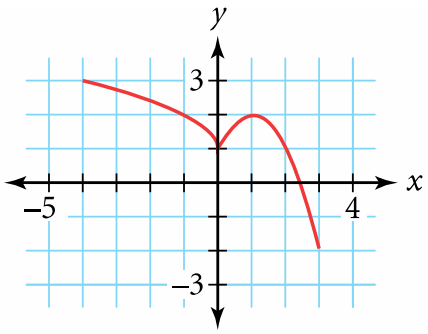
The curriculumwas based on *Discovering Advanced Algebra* byMurdock, Kamischke, &Kamischke which is designed to introduce multiple topics in parallel. For example, the graph of $y = \sqrt{x}$ is viewed as a "parent function" which is then used as a central example for demonstrating reflections of graphs across lines. This particular function is ideal for studying reflections across vertical lines because of the asymmetry of its graph. The instructor introduced the concept of piecewise-defined functions by giving examples combining the square root's graph along with a linear graph. This was done to provide a rich source of examples of how pieces of graphs can fit together.

There were nine slides in the lesson. The first slide introduced the graph of $y = \sqrt{x}$as the parent function and was used to illustrate transformations. Students were prompted to discuss the domain and range of the square-root function. The next slide defined the reflection of a function as a transformation that flips its graph$y = f(x)$across a line, creating its mirror image. The definition included the horizontal reflection across the *y*-axis, $y = f(-x)$, and the vertical reflection across the *x*-axis, $y = -f(x)$. The third slide explained a piecewise-defined function as "a function that consists of two or more ordinary functions defined on different domains". The students were prompted to work in groups to graph the following example from a slide:

$$f(x) = \begin{cases} 2x, & \text{for } -3 \le x \le 0 \\ \sqrt{x}, & \text{for } 0 < x \le 4. \end{cases}$$

The next two slides showed the students how to graph the function above one piece at a timeby first showing the $y = 2x$asa solid line where the function was defined; and as a dashed line as it continued on either end. The square root part of the graph was then included to complete the picture.

The next example reversed the thinking process of the previous problem. The students were provided with the following graph of a piecewise-defined function and were instead asked to find formulasdefining it.



It was noted that both pieces of this graph were reflections---one across the *y*-axis andone across the *x*-axis---of previously encountered parent functions. Students were prompted to determine which parent function was reflected across which axis. Next, the students were asked which translations they would need to perform to the reflected parent functions in order to achieve the desired graph. Students then worked in their groups to find the required equations. This was followed by a discussion of the solution, again presented on the slides.

The last slide was a summary of the transformations learned in the class, pointing out that a horizontal reflection across they-axis is defined as $y = f(-x)$, and a vertical reflection across the *x*-axis is defined as $y = -f(x)$.

Assessing and Observing the Lesson

Essential to any good lesson study is a well-designed assessment tool. Our assessment consisted of two parts: an attitudinal survey and a quiz on the lesson content. During our second meeting we critiqued the assessment and questions about the lesson that the course instructor provided. This took approximately one hour. We had considerable discussion on about the scope of individual quiz questions. For instance, care was taken in selecting the input values for evaluating the piecewise-defined functions in Question #2 of the quiz.

The attitudinal survey was adopted from the Fennema-Sherman Math AttitudeScales (Fennema& Sherman, 1976). Students were asked to rate each of the following statements on a five point scale from strongly agree to strongly disagree.

- *The lesson was clear.*
- *I understood the lesson.*
- *The pace of the lesson was appropriate.*
- *The use of visual aides (power point) was helpful.*
- *The work done on the board was helpful.*
- *The explanations by the instructor were helpful.*
- *I think I am good at math.*
- *I enjoy math.*

The quiz problems were carefully selected from the relevant exercises in the text.

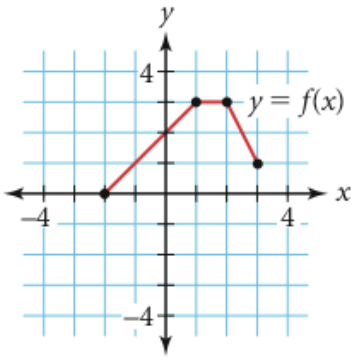
1. Let $f(x)$ be defined as the piecewise function graphed below.

Find each value.

a) $f(0)$

b) x when $f(x) = 0$

c) x when $f(x) = 1$



2. Let $g(x)$ be defined as

Find each value.

c) $g(1.8)$

d) $g(2)$

e) $g(4)$

f) $g(6.99)$

$$g(x) = \begin{cases} 3 & 0 \le x \le 2 \\ 2 + 0.5(x - 2) & 2 < x \le 4 \\ 2 - (x - 4) & 4 < x \le 6 \\ 1 & 6 < x \le 7 \end{cases}$$

The three observers were seated in the back of the classroom during the lecture portion of the class. During group activities, the instructor monitored student progress and assisted each group as needed. While the observers were free to roam around the classroom to observe how the students worked together, they made every effort not to interact with the students. At times the students would look to the observers for assistance. Whenever this occurred, the student would be politely reminded that the observers were not supposed to help.

Less than a week after the lesson, the group met again to discuss their observations and the results of the assessment. We also used this as an opportunity to brainstormnewwaysto teach the key concepts of the lesson. One suggestion was to use incometax brackets as initial motivation for piecewise-defined functions.

The observers agreed that there was a positive classroom environment. One observer noted that groups in the front of the class seemed more engaged. Potentially, more engaged students will sit up front and will have better class discussions while less engaged students may elect to sit in the back. The course instructor pointed out that strong students did in fact sit in the back. Perhaps some students who sit in the back are more prepared but less engaged. Overall, it was clear that the students responded positively to group work.

We felt it was important to adhere to the view that lesson study is not about critiquing the instructor; rather, the focus should be on observation of student learning. However, there was the opportunity to give some feedback. For example, it was observed that the instructor did not notice a student with their hand raised. While minor, it was a good reminder to her of the importance of paying careful attention to the classroom while interacting with individual groups.

Some comments from the observers were:

- *Lesson study provides a unique opportunity to observe the students in the classroom as opposed to observing and critiquing the instructor.*
- *Observers can gain unique insight as to how students respond in an active learning environment.*
- *It was good to have students explain their answers.*

Next we considered the survey results in Table 1.Unfortunately, there did not appear to be any correlation between attitudes and quiz results.

Table 1: Results for the Lesson Survey (n=24)

	Mean	Agree/S. Agree
lesson clear	4.00	71%
understood lesson	3.63	58%
appropriate pace	4.04	75%
visual aides helpful	4.38	88%
board work helpful	4.46	83%
instructor explanations helpful	4.13	79%
I think I am good at math	3.54	54%
I enjoy math	3.08	42%

The comments about the lesson itself were quite positive. The survey results showed 71% of the students thought the lesson was clear, the pace was appropriate (75%), the explanations provided by the instructor (79%) and the work done on the board was helpful (83%).The PowerPoint slides were seen as helpful (visual aids 88%) too.This is quite positive, especially the higher rated board work (83%) and visual aids (88%).

Unfortunately, the students had a more negative view of their own understanding of the content. Only 58% of the students agreed or strongly agreed that they understood the lesson. This is not encouraging especially given the amount of thoughtful work the group of instructors put into the lesson. Also discouraging, but not surprising was that only 42% of students stated that they enjoy math and 54% stated they think they are good at math.

Students had the opportunity to provide written comments that showed a wide range of responses from understanding to confusion about the lesson. The comments received were:

- *It was an informative lesson that greatly helped me understand a topic that I had no previous practice with.*
- *I think this lesson went well but it's something new and different so it was harder to understand.*
- *Thoroughly explained at a pace for students to absorb the information.*
- *Good review for me, very helpful in terms of reminding me of how to write horizontal and vertical reflections.*
- *This stuff is not clear at all to me. Could do more on it.*
- *I really don't get this lesson at all.*

While the students and the observers thought this was a successful lesson, the quiz results showed otherwise. The results were consistent with the students' negative view of their own ability. The mean score was 60% with a large standard deviation of 29%. On the quiz the overall pass rate (defined as a score of 70% or better) was 58%. (Ironically, this is the same percentage of the students who self-assessed as understanding the lesson.) Note, the University of Wisconsin's pass rate for College Algebra (defined as a grade of C or better) was 57% in 2009. Students performed poorly on finding the x-value when given the$y$-value in a picture of the graph (#1c) and in evaluating the piecewise functions at the boundaries most likely due to understanding the difference between inequalities and strict inequalities (#2d & 2e).

Table 2.Results for Quiz on the Lesson(n=24)

Problem #	**% Correct**
1a	79%
1b	67%
1c	52%
2c	63%
2d	52%
2e	42%
2f	63%
58% success rate (≥70%)	

Discussion

The lesson study was limited by the fact that it was done early in the semester and that the topics were challenging---the material was new for many of the students. The quiz was not as well aligned as it could have been with the lesson, which may have caused more difficulties for some of the students. Further, we did not specify the order that the assessment and quiz should be taken. Potentially, their perceived performance on the quiz may have influenced their responses on the survey.

Ultimately, our goal is to improve student performance. Because the assessment scores were low in the lesson study, the lead instructor decided to make significant changes in the structure of the class. The first major change was to make the PowerPoint slides available to view before class---"flipping" the curriculum. This allowed the instructor to discuss and summarize the content of the slides, providing more time to allow the students to practice problems in groups in class. The instructor was then able to help individual students more effectively. Also part of the new design was to have all the students write solutions on the board that the instructor then discussed and critiqued with the class.

The lead instructor gave a similar survey and quiz later in the semester to assess this new methodology. While it was a different lesson, and probably a more familiar topic, there was a marked improvement in the scores. For the survey, it was revealed that 100% of the students agreed or strongly agreed that working in groups and writing problems on the board was beneficial. Doing practice problems and having the instructor discuss the problems was similarly rated at 96% helpful. Other attitudinal statements that were used from the lesson study ranked higher as well. Further, the enjoyment of math improved by 14%, but was still only 56% overall.

A real improvement was seen on the quiz, where 80% of the students now passed, up 22% from the previous quiz. This was a significant gain and was an encouraging signal to the instructor that the pedagogical changes she made were having a positive effect. In the second lesson, the assessment began with the survey followed by the quiz to insure consistent responses.

In conclusion, the important outcome for our group was the ability to collaborate andto discuss and reflect on our teaching and our students' learning. We valued this opportunity to observe our students and to exchange ideas about how to teach these topics. With the goal of improving student understanding, enjoyment, and performance in math, we feel we have made some improvements in these areas.

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