Transmission of External Load from Its Point of Application through the Members of a Warren Truss to Its External Supports

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Abstract

Analysis of forces in each member of a wooden Warren truss has been presented in this paper using the method of joints. Reactions at the supports were calculated, direction of the force in the entire member was assumed as tensile or compressive. A joint where only two members are unknown was selected followed by joints on which three or more forces are acting and Lami's theorem was applied on that joint. The three equilibrium conditions were used ($\Sigma V = 0$, $\Sigma H = 0$ and $\Sigma M = 0$). Resolution of forces method was applied on the joint on which four, five, e. t. c. forces are acting. The result was used to draw a force summary diagram which shows that some of the members are in tension while others are in compression. The knowledge of this force transmission through the members will help in safe design of structures.

Keywords: truss, members, tensile, compressive, joints, forces, reactions.

1. *Introduction*

A truss is a structure made up of several bars or members riveted or welded together. Or,

A truss may be described as a structural framework consisting of straight individual members, all lying in the same plane and so connected as to form a triangle or a series of triangles. The triangle is the basic stable element of the truss. It may be readily observed that practically all trusses are composed of members placed in triangular arrangements, although some specialized trusses do not have this configuration Leonardo and George (1995).

The truss form emerged from the timber framing methods of classical antiquity in the Mediterranean region and only during the last two centuries became shaped by engineering analysis and design. Truss construction has always been associated with the high end of vernacular carpentry; trusses are rarely found in private homes or barns, but almost always in prestigious public buildings such as temples or churches, or in bridges. While we have only a small body of evidence for the exact form of the trussed roofs of antiquity, we have abundant extant examples of long-span roof systems from the Middle Ages through the Renaissance. The variety of forms and the inventiveness of their framers seem without end. Many of these pre-modern roof frames are fully realized trusses with a captured kingpost hanging the middle of the tie beam, and the ends of the rafters restrained within the same tie.

Trusses are used in a variety of applications in structural engineering and generally consist of a collection of members that are combined to form a structural component to resist loads. Most trusses form a flexural system in which the joints are idealized as pinned. Therefore, if the loads are applied at the joints (nodes), the members of the truss will only be subjected to tension or compression. Trusses are commonly used in long-span applications where efficient systems compared to girder elements can be fabricated to satisfy the structural demand with either constant or variable depth along the length. According to Rangsan, (2011), trusses have been used in several applications such as bridges, buildings, stadiums, or tower structures.

Wood trusses with spans up to and exceeding 80 feet are frequently utilized by post frame builders and have become a common feature in other types of construction.

Although long span trusses have performed very well in service, improper or nonexistent bracing has resulted in a disproportionately greater number of long span truss collapses during construction. In many cases, this is simply because many builders are not aware of the substantial increased susceptibility of longer trusses to collapse when they are not properly braced David, (2001). Wood gussets usually have sufficient stiffness and cross section to carry compressive loads from truss webs, without approaching a critical buckling load.

Roof systems used in residential construction have an outstanding record of structural performance Wolfe, et al (1986). Truss roof assemblies are currently viewed as a collection of individual trusses connected by a load-distributing element Wolfe and McCarthy, (1989).

The trusses considered in this paper are planar trusses; that is, all of the truss elements and all of the applied loads or forces lie in the same plane. Further, all loads are applied at points of intersection of the members and the truss members are assumed to be connected at their points of intersection with frictionless hinges or pins; in effect, they permit the ends of the member the freedom to rotate. Because the ends of the members are assumed to be pin connected, the members must be arranged in the triangular shape if they are to form a stable structure.
As shown in Fig. 1 structures of four or more sides that are connected with frictionless pins at their points of intersection are not stable and will collapse under load.

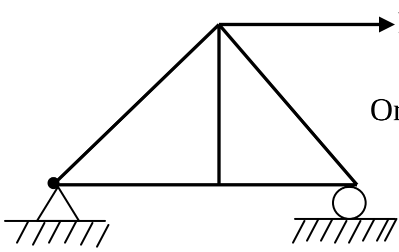

Fig. 1a Rigid, Stable Framework

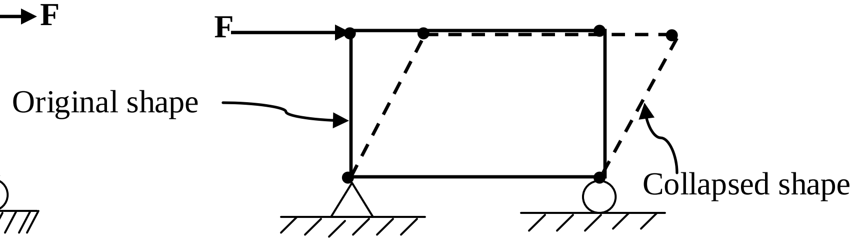

Fig. 1b Non-rigid, Unstable Framework

Trusses are fabricated units and may be considered to be very large beams. When loads are extremely heavy and/or spans are very long, normal beam sections will not be adequate and trusses will be used. Most trusses are constructed of either metal or wood.

As compared to a solid bending member, trusses are generally economical with respect to material: however, fabrication costs are high. The applied load in trusses is transformed into the axial force in the members. Although some bending is induced in the truss elements, the magnitude is usually negligible compared to the axial force component Rangsan, (2011). The application of stress to any material will lead to the production of elastic and / or plastic strain and if the stress is increased progressively, fracture will ultimately occur Raymond,(1990). Due to large numbers of examples of compound stresses met within engineering practice, the cause of "failure" or permanent set under such conditions has attracted considerable attention Rajput, (2010). When all the forces that act on a given part are known, their effect with respect to the physical integrity of the part still must be determined Leonardo and George (1995).

The objective of our analyses is therefore to determine the forces that are developed in the various members of the structures and how the forces from the point of application of external loads are transmitted through the various members of a warren truss to its external supports

2. Materials and Methods

The analysis process involved the analytical application of the conditions and laws of equilibrium for coplanar forces.

The type of truss used for this paper is a perfect warren truss whose dimensions are given in the schematic diagram shown in figure 2. The truss is pin-connected at point A and roller at point E. It is considered perfect because it is made up of members just sufficient to keep it in equilibrium when loaded without any change in its shape and it is a triangle, which contains three members and three joints puts together.

It is also rigid in nature because there is no deformation on application of any external load. It also obeys the law;

n = 2j – 3 and therefore statically determinate

Where n = number of link or member;

j = number of joints.

Method of Joints is used for the analysis.

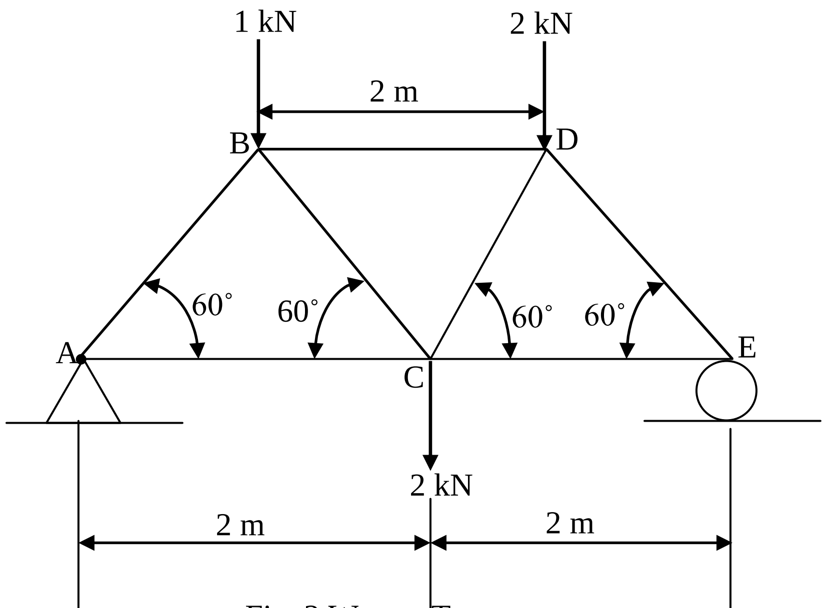

Fig. 2 Warren Truss

For the analysis of a truss to be simplified, the following assumptions are made:

- All members of the truss lie in the same plane.
- Loads and reactions are applied only at the panel points (joints) of the truss.
- The truss members are connected with frictionless pins.
- All members are straight and are two-force members; therefore, the forces at each end of the member are equal, opposite, and collinear.
- The line of action of the internal force within each member is axial.
- The change in length of any member due to tension or compression is not of sufficient magnitude to cause an appreciable change in the overall geometry of the truss.
- The weight of each member is very small in comparison with the loads supported and is therefore neglected.

2.1 The Method of Joints

Steps used for method of joints:

- Calculate reactions at the supports.
- Make the direction of the force in the entire member as tensile. If on solving the problems any value of the force comes to negative, that means the assumed direction is wrong and that force is compressive.
- Select a joint where only two members are unknown. First select that joint on which three or less than three forces are acting. Then apply Lami's theorem on that joint.
- Draw the free body diagram of the selected joint since whole truss is in equilibrium, therefore the selected joint will be in equilibrium and must satisfy the equilibrium condition of the coplanar concurrent force system. I.e. $\Sigma V = 0$ and $\Sigma H = 0$.

- Now select that joint on which four, five, e.t.c. forces are acting. On that joint, apply resolution of forces method. Note, if three forces are acting at a joint and two of them are along the same straight line, then for the equilibrium of the joint, the third force should be equal to zero.

3. Results and Discussion

The Free-body diagram of the truss is shown in Fig.3

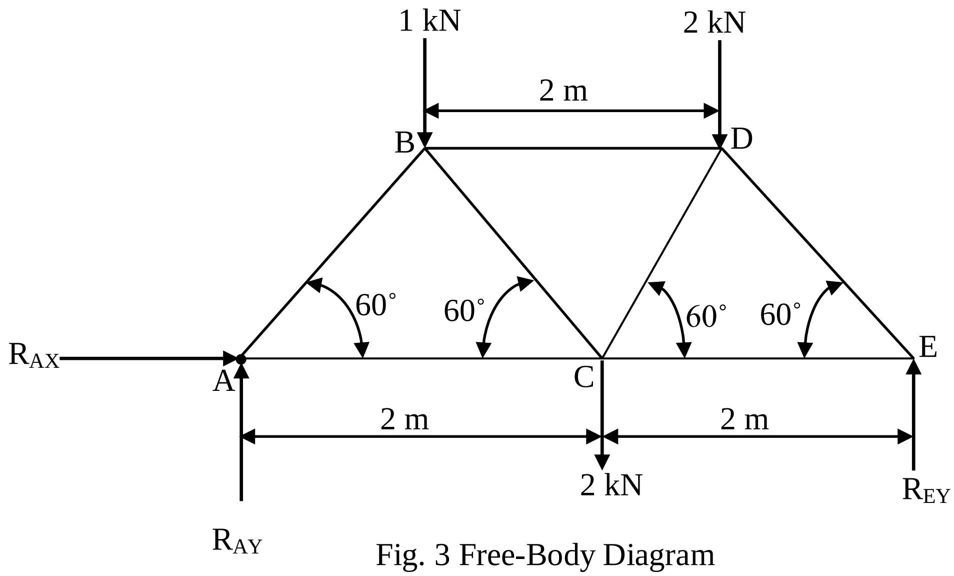

Fig. 3 Free-Body Diagram

The three equilibrium conditions were used:
ΣV = 0..(i)
ΣH = 0..(ii)
ΣM = 0..(iii)

The truss must be in equilibrium. Therefore, the reactions at A and E will be calculated first. Taking counter clockwise moments as positive, sum moments about point E, we have,

$$\sum M_E = -R_{AY}(4) + 2000(1) + 1000(3) + 2000(2) = 0$$

from which R_{AY} = 2250 N

Taking moment about point A,

$$\sum M_A = R_{EY}(4) - 2000(2) - 2000(3) - 1000(1) = 0$$

from which R_{EY} = 2750 N

Note that there are no horizontal forces or horizontal components of diagonal forces. Applying $\sum F_X = 0$, this implies that R_{AX} = 0.

Now that we have calculated the truss reactions, the internal forces in all the truss members can be computed. Each joint will be isolated in sequence as a free body. Since a joint with more than two unknown forces cannot be solved completely, a logical joint to start with would be joint **A** or joint **E**. The free-body diagram for joint A is shown in Fig. 4(a). Let the unknown forces be designated as F_{AB} and F_{AC} and should be interpreted as the forces in members AB and AC. The lines of action of the two unknown forces are known, but the sense and magnitude of each are unknown. If the sense of an unknown force is not obvious, it must be assumed. A negative result would indicate that the sense was opposite to that assumed.

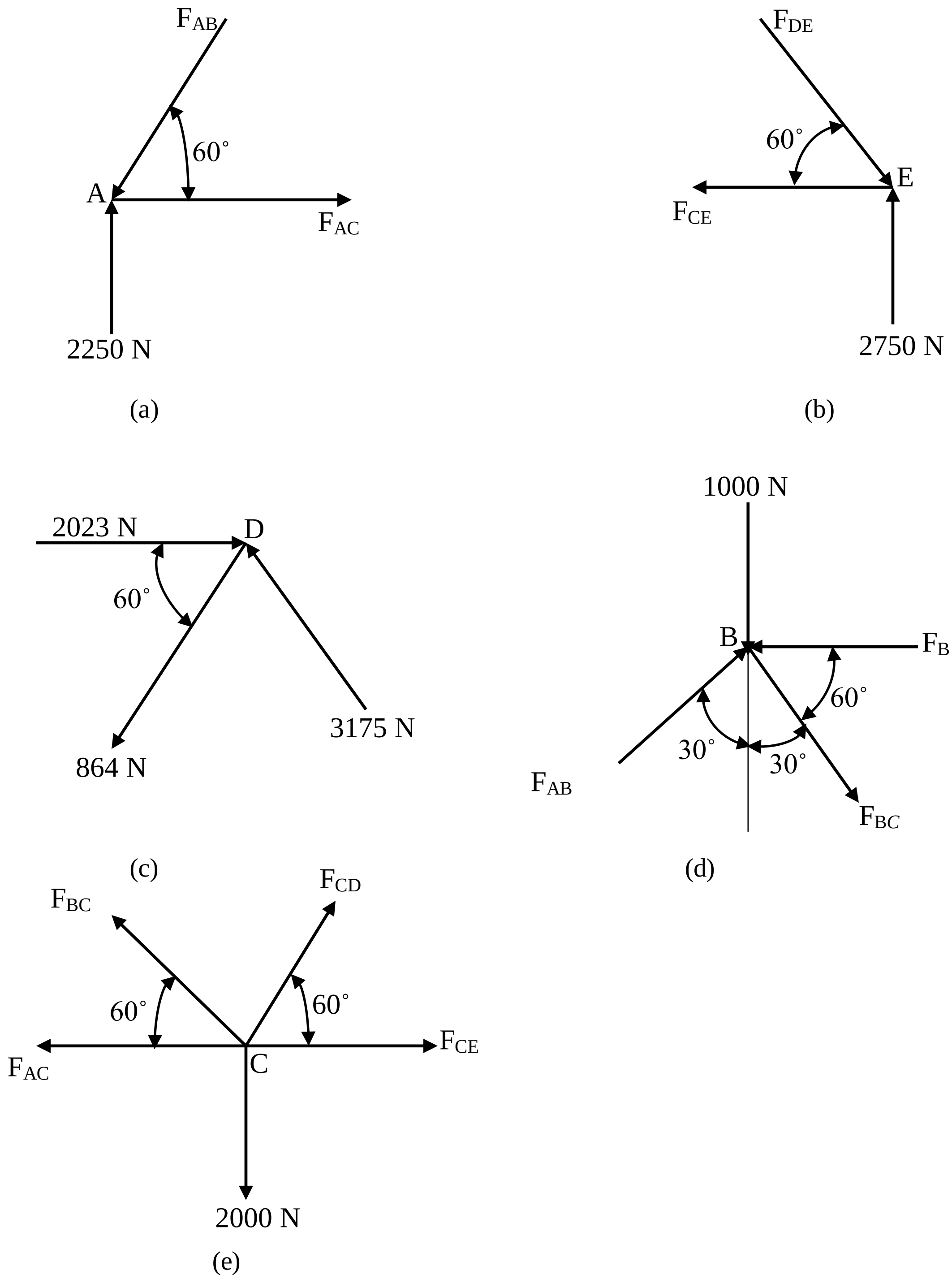

Fig. 4 Isolated Joints of the Truss

In Fig. 4(a), force F_{AB} is shown pushing into the joint (i.e. acting toward joint). This means that member AB is assumed to be in compression and is pushing into the joint. Force F_{AC} is shown acting away from the joint. Member AC therefore is assumed to be in tension, pulling away from the joint.

Using vertical and horizontal coordinate axis system with upward-acting forces and forces acting to the right as positive, first sum the vertical forces

$$\sum F_Y = R_{AY} - F_{AB}\sin 60^\circ = 0$$

= 2250 − F_{AB} (0.866) = 0

from which F_{AB} = 2600 N (compression)
Summing horizontal forces (i.e. $\sum F_X = 0$) we have

$$\sum F_X = F_{AC} - F_{AB} \cos 60^\circ = 0$$

= F_{AC} − 2600 (0.50) = 0 from which
F_{AC} = 1300N (tension)
Since the results are positive values, the senses for forces F_{AB} and F_{AC} are as assumed. (Member AB is in compression and member AC is in tension).

Consider joint B. The free-body diagram is shown in Fig. 4(d). There is one known force (F_{AB}) and two unknown forces (F_{BD} and F_{BC}) as well as one external load of 1000 N. Definitely, force F_{AB} is known to be compressive (as was determined at joint A) and is indicated as acting into the joint. The 1000 N is shown acting directly on the joint. The lines of action for forces F_{BC} and F_{BD} are known, but their senses and magnitudes are unknown. Assume member BC to be in tension member BD to be in compression as shown in Fig. 4(d).
Summing forces in the Y-direction,

$$\sum F_Y = -1000 - F_{BC} \cos 30^\circ + 2600 \cos 30^\circ = 0$$

from which F_{BC} =1 445 N (tension)
Summing forces in the X-direction,

$$\sum F_X = -F_{BD} + 2600 \sin 30^\circ + F_{BC} \sin 30^\circ = 0$$

= − F_{BD} + 2600(0.500) + 1445(0.500) = 0
from which F_{BD} = 2023 N (compression)

The positive results indicate that the senses for forces F_{BC} and F_{BD} are as assumed. The member BC is in tension and member BD is in compression.

Now let us consider joint C. The free-body diagram is shown in Fig. 4(e). There are two known forces (F_{AC} and F_{BC}), two unknown forces (F_{CD} and F_{CE}), and one external load of 2000 N. The two known forces, F_{AC} and F_{BC}, are indicated pulling on the joint, since from our previous computations at joint A and B, both were found to be tensile forces. Members CD and CE are assumed to be in tension as shown in Fig. 4(e). Summing vertical forces (i.e. $\sum F_Y = 0$), we have,

$$\sum F_Y = -2000 + 1445 \sin 60^\circ + F_{CD} \sin 60^\circ = 0$$

from which F_{CD} = 864 N (tension)
Summing horizontal forces, (i.e. $\sum F_X = 0$), we have,

$$\sum F_X = F_{CE} - 1300 + F_{CD} \cos 60^\circ - 1445 \cos 60^\circ = 0$$

$$= F_{CE} - 1300 + 864(0.500) - 1445(0.500) = 0$$

from which F_{CE} = 1591 N (tension)
Since the results are positive values, therefore, the senses for forces F_{CD} and F_{CE} are correct as assumed. Both members are in tension (i.e. member CD and CE). Consider either joint D or E. The last remaining member force to be computed is that of member DE. Select joint E to analyze. The free-body diagram is shown in Fig. 4 (b). Here, only force F_{DE} is unknown. Assume member DE to be compression, then applying the equilibrium equation i.e. $\sum F_Y = 0$, we have,

$$\sum F_Y = 2750 - F_{DE} \sin 60^\circ = 0$$

from which F_{DE} = 3175 N (compression)

The positive sign signifies that the sense of force F_{DE} is as assumed. Conclusively, member DE is in compression. As a means of summarizing the forces in the truss members, a force summary diagram, is shown in Fig.5. The force in each member is designated T or C for tension and compression respectively.

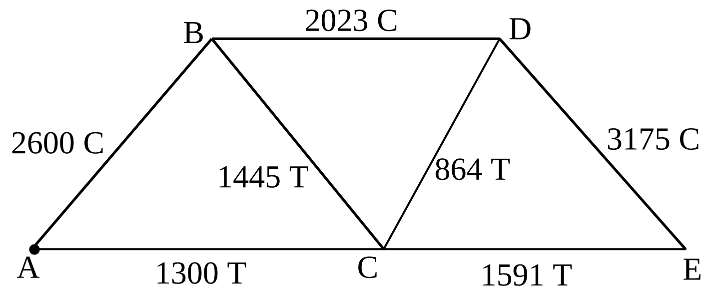

Fig. 5 Force Summary Diagram

4. Conclusion

The forces that are developed in the various members of the structures and how the forces from the point of application of external loads are transmitted through the various members of a warren truss to its external supports have been determined with their senses. Figures 2 and 3 show the warren truss and the free body diagram of the truss respectively, figures 4 (a-e) show the isolated joints and figure 5 shows the force summary diagram which shows that some of the members are in tension while others are in compression.

The truss has been broken down into five joints (A – E). The method of joints used consists of removing each joint in a truss and considering it as if it were isolated from the remainder of the truss. A free-body diagram of the pin is the basis of the approach. The free-body is prepared by cutting through all the members framing into the joint being considered. Since all members of a truss are two-force members carrying axial loads, the free-body diagram of each joint represents a coplanar concurrent force system. Since the truss as a whole is in external equilibrium, any isolated portion of it must also be in equilibrium. Therefore, each joint must be in equilibrium under the action of the external loads and the internal forces of the cut members that frame into the joint.

The equilibrium equations ($\sum F_X = 0$, $\sum F_Y = 0$ and $\sum M = 0$) are used to determine the unknown forces in a coplanar concurrent force system and the reactions at the supports. It was shown that when using the method of joints, no more than two unknown member forces can be determined at any one joint. Once these unknown forces have been calculated for one joint, their effects on adjacent joints are known. Successive joints are then considered until the unknown forces in all members have been determined. The knowledge of this force transmission through the members will help in safe design of structures.

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