Stochastic Volatility and Black – Scholes Model Evidence of Amman Stock Exchange

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Abstract

This paper is an attempt to decompose the Black – Scholes into components in Garch option mode, and path dependence of the terminal stock price distribution of Amman Stock Exchange (ASE), as Black – Scholes the leverage effect on this paper result of analysis is important to determine the direction of the model bias, a time varying risk, may give a fruitful help in explaining the under pricing of trade stock shares and traded options in ASE. Generally, this study considered various pricing biases related to warrant of strike prices, time to time maturity. The Garch option price does not seem overly sensitive to (a, B_1) parameters, or the time risk premium, variance persistence parameter, $\Omega = a_1+B_1$ heaving on the magnitude of the Black –Scholes bias of the result of analysis, where the conditional variance bias doesn't

Keyword: Black-Scholes, Stochastic volatility, ASE, Garch model, Option prices, Strike prices.

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Section One: Introduction:

1.1 Definition terms of the study:

We should clarify some terms which used in this study, such as:

- An option: it is the right for the dealer to buy or sell goods at a strike price, but not obligated.
- Exercise price or strike price: it indicates to a price that which good in particular may be bought or sold inuring a specified period or before,
- In the money: this related to immediate price, which has positive intrinsic value. In addition, it gains from the immediate exercise.
- At the money: in this expression, there is on equalization state between the exercise price and share price.

1.2 Objectives of the study:

This paper aims to investigate the ASE stock and strike prices and the other sub – objectives are:

- We can use the modification of Black Scholes equation and option pricing also can use in this field.
- With respect of market value when testing the Black –Scholes equation on the basis of ASE index and market value stochastic volatility present strongly.

1.3 Research question

The main question raised is, whether the prices that the dealer has paid s in the ASE market to buy or sell is fair prices, and whether these prices can be valued by Black – Scholes equation Then other questions are as stated as:

- : Is Black Scholes model reasonably accurate for in –the money pricing of stock shares in ASE?
- : Does the Black Scholes model appropriate for pricing the traded warrants on the ASE.
- : Can the investor who have little grasped and understanding of the nature of option warrants rarely make good profit?

1.4 Brief notes of ASE

ASM (Amman stock market) or some named it Amman stock exchange (ASE), ASE has proposed to have an increased of companies which are listed throughout the study period ,which can give us assigns that there is a confidential trust of ASE regulations ,and trust of economic growth of Jordan, in addition to these this let us have a positive signs of economic directional growth and stability on a country the improvement of ASE is demonstrated like any emerging market on its characteristic , the ratio low turnover ratio, low liquidity, low transparency, and the non existence of market decision makers, the turnover ratio for the period of the study was 17.53%, and the daily average of turnover was 0.9593 %, this ratio is too few, and these ratios are considered to vary per time. , The major actions that may affect shares buy and sell and trading activity mainly is prices. And the average daily turnover is ownership of individual investors and institutional, and government. The ownership structure The ASE has witnessed an increased in the number of listed companies throughout the years, which gives an indication of economic growth in Jordan, and stability during the period of 2003 -2010 in ASE.

The trading volume increased year to year during the period of study, the results of visibility of ASE is superior than other stock markets in middle east region, it has undergone accelerated growth, especially during the last 6 years due to stability and the Arab shares such Iraqi an investor, also Jordan government represented the board of international accounting standard. Some indicators of ASE, It established 1976 and it is emerging stock market, the capitalization is 9.765m us, and the change 1n 1999-2001 is 8.4%, while it in 1996-2010 is 27.3%, where the capitalization ratio to GNI IS 58.9%, where the turnover is 13.54 mus, and the turnover (liquidity)is 18.7%). (M.Alaya, 2013).

1 – 5: Previous studies of the subject:

(Whaley et al. 1987), they have studied, and analyzed the CBOE data for limited period extended from 1975 to 1978, they have used time of expiration date, and the strike price, their results indicate that the volatility eliminated, and concluded the dividend divided induced probability integrated in American call option pricing. Brown has observed a random behavior of option pricing, he noticed the motion of pollen floating in water is randomly, and did not follow any distinct pattern, this observation and it is subsequent is known as Brown motion..Fischer black and Myron scholar employed these tools in their research to model to price derivative of securities. The Black-Scholes formula used to price European call and put options based on asset price of dynamic stochastic model, which depend on a stochastic differential equations, therefore we can solve this model of geometric Brown motion, where the certain two real parameters of volatility are drifting. The use of statistical tests to solve the problem associated with the calibrating stochastic dynamical system. (Fisher Black and Myron Schools, 1973). We can define options as: security has rights of buy or sell in certain conditions, with a limited and specified period.

The simple type of option is one that gives the right to buy or sell a single share of stock, this called call option. (Black-Scholes,1973)., They noticed that if an option has a higher price daily traded value in the stock market, then the option has a high traded value, then the option should be exercised, Also when expiration date is near or closed to, there is an equilibrium between the value of the option or stock share and the price of the stock.(Dumas et al. 1996,1998), they have studied s & p 500 index option data, through the period 1988- to 1993. They have evaluated the volatility, also they have found that performance is worse when we compare to the Black – Scholes model. Were (Sorin-Staraga, 2004), used a sample of s & p 500 index option prices, utilized of option prices and implied of deterministic volatility approach to his data, results indicate that a parsimonious model is suitable for his analysis through Akaike criterion, and the model predicts the errors which grows largely as volatility function, and hedge ratio determined volatility by Black – Scholes equation.

The Black –Scholes model has been still used in option pricing, where the input is important required for the call option pricing, on a non dividend option. Hull and White (1987 referred to as How, provide throng a power series, which compared and this How model can provide two models with stochastic volatility. Rubinstein (1994), he proved the volatility of asset returns and option, which can be appear as a deterministic function, then he discussed the behavior system of changes of. Brande, et al. (2002).

Find that the implied binomial model poises the American style options. Were Cox. (1979). Applied a tree model of volatility. Some authors using in their studies ad- hoc procedure of Black-Scholes implied volatility. The volatility increases the probability that the stock price will rise or fall increases. This study is arranged as follows: Section One: will include objective of this study. And Assumptions of Black-Scholes model, then the hypothesis of study. Where section two introduced the background and theoretical review of black-Scholes model, it will also discuss the model that this study follows. Sections three: contain the pricing equation derivation. Section four: included an overview of data sources and methodology. section five included the results and empirical results of this paper, finally we concluded the final results of the study.

Section two: Literature review:

2 – 1: Assumptions of Black-Scholes model:

The black-Scholes model based on the following assumptions:

- 2 The interest rate is known and constant through time in a short term period.
- 3 The stock pays on dividend or other distributions.
- 4 The stock price follows a random walk in continuous time with a co- variance rate proportional to the square of the stock price.
- 5 There are no transaction costs in buying or selling in the stock or options.
- 6 Then are no penalties for short selling and the seller who does not own security will simply accept the price of the security from a buyer (Black & Scholes, 1973).
- 7 The underlying asset follows a long normal random walk.
- 8 The Arbitrage arguments allow us to use a risk-neural valuation approach (Cox-Rabinstein's Prioof).

Black-Scholes in their paper derived on the option prancing model, of which one of main assumption was that underlying stock follows a Geometric Brownian motion. Black-Sholes in their paper exists many ways to drive stock option prices, many approaches can be used such as a martingale approach which depends on risk-neural valuation formula as:

Where V_{op} : option payoff at maturing $V_{pay} = \eta$ (K, ST) is some deterministic function and V_{op} is option value

as of time t = 0,, $\eta assets_t$ is a so called numeric asset used the relative price $\frac{V_{pay}}{\eta asset_t}$ (2)

The Black - Scholes model assumes there is no arbitrage opportunity, which emphasizes that there is an opportunity to gain profits, without any risk involved. The arbitrageurs themselves be aware of it by buying more there in one stock market, which can be causes the price of the stocks to rise there, and selling in other stock market, which causes the price to decline, there such as buying more stock in the New York stock market, and when there is a movement to raise up prices selling there in London stock market. The Black - Scholes model for option pricing divide with the idea of delta hedging in mind. The Black-Scholes equation is widely soon to have paved the way for an influx of mathematics of finance and financial studies, the pricing formula can be expressed as Black-Scholes for put option as:

$$f(S,T,K,r,\sigma) = Ke^{-rt}N(-d_2) - SN(-d_1)$$
(3)

Where: S is the current stock price, T is time until expiration in years, and σ is the annualized volatility of the stock, N is the cumulative standard normal distribution function, r is the current risk-free rate of return. The above function inputs required for the pricing of a call option on a non-dividend paying stock with current stock price strike price, interest rate, volatility and time to maturity. We can observe all these parameters. Through above discussion the following alternative hypotheses are expressed as:

H(1): The market value for in the money warrants against Black-Scholes value for in the money warrants also for out-of-money warrants.

H(2): The market value of of for near-maturity warrants.

 $_{H(4):}$ The market value on all daily prices observations are also volatile time to time. H(5): Trading in stock takes place continuously and market are always open;

H(6): The stock pays no dividend on any type of underlying assets or security;

H(7): The stock price follows a Geometric Brownian motion process, as constant.

H(8): The assets are completely divisible in nature;

H(9): There is no penalties on short selling of shares and investor also get full use of short-sell Procedure; and Stock price option follows an explicit type of stochastic process called diffusion process; With the exception of volatility in the market we can express d_1 and d_2 in the (4) equation as:

 $^{2}\sigma$: is the variance of courteously compounded return. Stochastic processes have many properties that appear in numerous application, such as:

- 1- The of $X_{ti} + h X_{ti}$ distribution depends only on h, then X_t Is have stationary increments.
- 2- if $t_1 < t_2 < t_3$, $X_{t2} X_{t1}$, $X_{t3} X_{t2}$, are independence, that sign off X_t , this gives us assign that the process with indecent increments.
- 3- As we discussed in this review X_t is said to be the Markov property due to the independent state of variables.
- 4- We can see that the transition probabilities should be normally distributed in function (5), which as follows:

$$P_{j}K = f(K, 2N, P_{i}) = \left(\frac{2N}{K}\right)P_{j}K(1-P_{j})^{2N-6}$$

Where $K \in (1, 2, \dots, 2N)$ and f is the probability mass

Function for the binomial distribution further more one can restrict his analysis by looking at stationary market chains.

22: Hedging portfolio through Black-Scholes equation:

If we start, our analysis one important note should be considered of driving the black-Scholes equation; via a binomial pricing process, which is based on the binomial price model described in the above section. First, we assumed one risk-free rate at which we can borrow or lend money and that there is one period left call option on the stock. Therefore the fair value of the option which we have determined to be by requiring that it be the value that equals it is expected payoff. This can be determined merely by knowing some information about interest rates, underling stock, the strike

price, and the range of the values that the stock can take on after a period and we should know the possible values that the stock could take over this period, which has a relative to what the volatility in a way in the Black-Scholes equation volatility of stock can be defined to be the standard deviation of the logarithm of the returns of the stock. A Black-Scholes model assumes a constant volatility and one way can be used to estimate is to use historical volatility. The equation of estimation can be given as:

Where: (n-1) price data from (n-1) and u_i is $\ln \frac{\delta i}{\delta_{i-1}}$, \overline{u} is the sample average of all u_i , δi is the stock

price in period t and T is the total length in each period in the year. T equal 1/252 or 1/365. The numerator represents the log-returns of the stock, and the denominator is a scaling factor to make the estimates be equal one of the year volatility. (Hull, et. al., 1987) wrote an article the pricing option on assets with stochastic volatilities to have a solution to the problem with a solution to the problem of pricing European earl options.

They determined that the price depends upon some stock price S, and it is instantaneous variance, $V = \sigma^2$. Function (7) can be written as:

$$d_{s} = \phi S d_{t} + \sigma_{s} d_{w}$$

Where $d_{v} = \mu d_{vt} + \delta_{v} d_{2}$

 ϕ : for the stock price may depend on S, σ ant t.

U: and the diflusion coefficient.

δ : for the variance which may depend only on σ and t.

The two processes are correlated. Hull and White, they determined some assumption according to their work, such as:

- 1- The volatility (V) is not correlated with the stock price.
- 2- The volatility V is not correlated with aggregate consumption.
- 3- S and δ are only the two variable which are effecting the price of derivative function, therefore the risk free rate must be constant or deterministic.

The expected returns of the stock for example, are not independent of risk preferences. Risk in stock market adverse investors in how he would ask for higher expected returns for increasing risk level and risk seeking investor d would ask for lower expected returns for decreasing risk levels.

Section three: Volatility Models:

In this model we can depend on historical returns from stock prices if we use the moving Average model the formula is:

$$u_i = \frac{S_{ti} - S_{i-1}}{S_{i-1}} \quad(8)$$

Where: u_i percentage changes of returns of the day i-1 and of day i-1 and the end day i, Si: represents value of

assets at day i., The value of the day before of u_i being $u_i = \ln\left(\frac{Si}{S_{i-1}}\right)$ the unbiased estimate – of one

volatility as:

$$\sigma_n^2 = \frac{1}{m-1} \sum_{i=1}^m U_{n-i} - \overline{U}^2 \dots (9)$$

The Stochastic model proposed some properties:

- 1- Implied volatility has a random behavior at time, where the smooth dependency is available..
- 2- The Implied volatility is more sensitive according to the Wiener processes, where the disturbing noise is normally distributed.
- 3- The market volatility is reduced,
- 4- Stochastic volatility model can be used simply to evaluate any stock market or portfolio.
- 5- The share and option or stock prices can be determined or given through the applied of Black-Sholes equation.

3.2 Option pricing model

Black-Scholes one of their attributes the lognormal distribution has that stock price can never fall by more than 100 percent, in other hand, there is a small chime to raise more than 100 percent, according to these the Black-Scholes model in one of their assumption weather the options and stock prices are determined through Black-Scholes equation lagged and normally distributed. Pricing derivative will provide a payoff at one exact time in the future simply because it takes a discounted vat is risk-free interest rate (Er_i) , from the underlying assets and discount vat is a risk free interest rate (r)..Recent studies finding shows that volatility tends to vary over time according to this situation the assumption of constant volatility is unrealistic. The Garch option model assumed that the conditional volatility of stock prices depends on the past pricing errors. Therefore, one phase of this study is Garch models, and the study based on historical return from ASE stock prices. Regarding to the nature of random variables which distributed as log-normal as a Markova process is a certain stochastic that may randomly vary, and on the other hand that history of variable prices irrelevant and the only present value is used to predict the future values. Brownian motion is a partial equation of Markov process that posse's variance 1.00 and zero mean as (John. C. Hull, 2006).

This method, known as a Wiener process variable (x) follows this method when:

1- ΔX : represents two short intervals at time t, where Δt indicates to the independent variable of the series. If we put equation in a simplified approach, so that:

 $\sigma_m^2 = \frac{1}{m} \sum_{i=1}^m U_{n-i}^2$ In this method we should be aware of the data too old might be unrelated to predict the

future, then we should carefully about more data which lead to better precision usually we used the chasing prices from daily data over "the last 90, 180 or even 252 days are often used.

2- Garch model: this method usually used by many models that follows the implied volatility as (Engle, 1982). According to this process, we should make the unconditional implied volatility as constant, where on the other hand the conditional implied volatility can be changed and vary over time(t), this model is known as an arch model (q) is:

Where ω is equal γ . V_L and is calculating by using the following equation:

$$u_i = \frac{Si - S_{i-1}}{S_{i-1}}$$
 or $u_i = Ln \frac{Si}{S_{i-1}}$ (11)

And is the long-term average of Co- variance and variance rate and γ is weighted assignment to Vi variable,

and S_{i-1} : is the order of dependency to past returns. An ArCh model as a generalized approach model was proposed in a paper of (Boterstev, 1986), which know Grach model (P, q), can be as:

$$\sigma_n^2 = \omega + \sum_{i=1}^{q} a_i U_{n-i}^2 + \sum_{i=1}^{p} B_i \sigma_{n-i}^2 \qquad (12)$$

Where: $\omega > 0$ $a_i \ge 0$ And. $B \ge 0$.

which (ΔX) during 3- The changes happen on а (Δt) is short period $\Delta X = \sum \sqrt{\Delta t}$ (where \sum disributed normally $\phi(0,1)$). But in a long time from (0) to maturity T, x (T)-X(σ) is normally distributed with mean zero, standard deviation \sqrt{t} and variance T the Wiener process for a variable C add an expected drift rate μ and volatility, which can be written as: $d_c = d_t + d_x$. The variable C is normally distributed in any time of the interval T, mean change of C is (μT) , standard deviation $\sigma \sqrt{T}$ and variance $(\sigma^2 T)$.

The stochastic implied volatility of ASE assets as a emerging market assets are randomly distributed, we can measure the volatility changes in a certain time of unexpected changes of assets and other financial assets, when the fluctuation can't be captured and unknown, therefore the implied volatility can measure the risk of options and assets. With different exercise and strike prices the volatility is skewed, and shows a skewed or smile sometimes instead of linear curve, in this case the smile is reflecting a higher implied volatilities for cash flow or deep in- or out of the money options. When the market price has used the implied volatility model and the stock price follows a stochastic process and as Arch model, variables (p,q) is the order of dependency the simplest model Garch model is Garch (1, 1) model, which can specify as:

$$\xi_{i} = \omega + Q.U_{i-1}^{2} + B.\sigma_{i-1}^{2}$$
(13), and B must sum to one.

(Engle and Bollerstev, 1986) have introduced the I. Garch (integrated Garch), in this model a,B is equal to I. when we like to estimate the parameters of the model we should have in our results the important results of maximum log likelihood estimation. The likelihood of the (m) observation as:

Which estimated variance for the day I and are the number of observations? The best parameters are the therefore the logarithm of the expression is equivalent, thus the log likelihood expressed as:

The parameters of Garch (1, 1) can be defined as a function of time, it means that the parameters at day (i) are estimated by means of maximum likelihood, and subsequently these parameters are used to forecast volatility at day (i), then we can adapt equation (13) as:

$$\sigma_i^2 = \omega_i + a_i \cdot U_{i-1}^2 + B \cdot \sigma_i^2 , \qquad (16)$$

 ω , a, b are changing over time v, V_L , U_{i-1}^2 and σ_{i-1}^2 change over time also, and the model estimated through daily maximization of the parameters, it's expected, which known as a geometric Brownian motion C (John, Hull, 2006) $d_v = \phi_v d_t + \zeta_v d_w$ where the (d_t), and (d_w), are correlated to be before to estimated future

volatility on daily basis.

Section four : Data sources and methodology:

4 -1 : data sources and collection : The ASE database and stock prices database for the period from January 2010-2015 considered in this paper analysis as historical data of volume of traded shares, price of option, price of underlying stock, strike price, expiration data and a time stamp that reveals at which time. The collected data studied were 2014, daily value of the ASE index analyzed as the figure(1) below, and the specified index data considered as the daily closing value of the index and the bid prices of these options during the period of the study. Figure(1): shows that the data on the ASE index from 2005 up to 2014 that volatility is clearly appears.

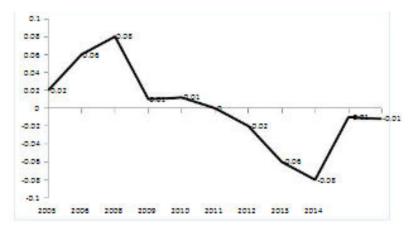


Diagram (1): Daily Returns of ASE Index from 2005-2014

B: Methodology: There are two methods of analysis the huge data amount first step I have used the (OLS), then implied volatility by Grach (1, 1). Section Five: empirical results :

	Minimum	Maximum	Mean	St/deviation
Volatility	0.02	1.98	8609	1.7976
Residual	6.73	15.489	3.164	0.9682
* Square residual	0.03	1.6313	4.728	3.6443
* Square relative residual	0.00	6.883	3.669	1.4865

* Squared relative deviation:

$$M(\hat{P}) = \frac{(\hat{P} - P)}{P} \quad (\hat{P}) \text{ ; Represents the price of Black-Scholes equation, and P is the actual stock price.}$$

We can summarize that the stocks have stochastic implied volatility, we have noticed that when we applied the measurement to the historical data, different types of volatilities are implied, this happens when stocks are over evaluated as (15.429), were the lowest strike price option is over valuated (1.64). These results persistent the bias which appears when we plot the volatility versus time, the average of ASE options overpricing by 31% with a low 17% under pricing. The (OLS) results of the analysis are shown in the following tables.

Table (2) Model Summary

Model	R(correlation)	R ² squired	Adjusted R ²	St/Error	D-Watson
1	0.161	0.259	0.213	1.8652	0.367

Dependent Variable Square Relative residuals

Table (3): Coefficient of OLS

Model Constant	Unstandardized Coefficients		Stand Coefficient	t-test	Prob.Level
Constant	В	Std. / Error	Beta		
Time	3.071	0.299		9.735	0.000
	0.019	0.001	0.263	2.594	0.57

Dependent Variable: Square relative residuals

The effect of time is well appear in the dependent variable chosen, this effect is positive significant at the 5% level, since the coefficient still positive. This can let us to conclude that increasing of time, which dealing with data, increasing results becomes as the pricing bias, insofar the short- run the options and stocks should minimize the pricing bias, this can be shown through Black –Sholes equation applied results which predicts the linear function of volatility and the residuals, then we can predict the linear function of prices.

Finally, it seems to be a common pattern therefore we can summarize that there is spikes which observed, this can be interpreted due to sudden changes in the option and stock prices. We can conclude that there is a positive auto correlation present in the analysis, this means that values of implied volatilities are correlated to each so we can conclude that in the absence of unusual spikes, give assign that best estimator of historical volatility are presented in our analysis.

The maximum likelihood Estimation Garch (1, 1) model: In table (4) the estimated represents and the log of Garch (1, 1) model of ASE index.

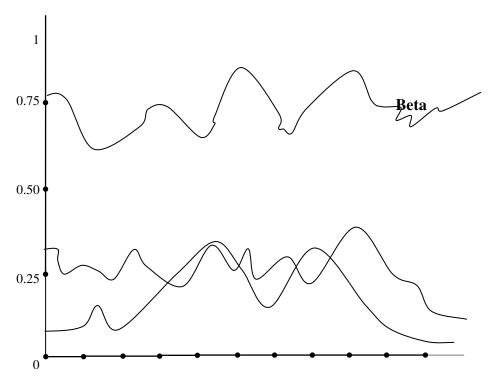
V _L	ω	γ	a	В	a + B
6.532E-0.4	1.263E-0.4	0.296	0.0367	0.6932	0.7299

Table (4): results	of Garch(1,1_	_)of ASE index
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Due to results of table (3) parameter α is 0.0367, B is 0.6932 and γ is 0.296 this means that past returns give us a sign that it proved a some information to estimate present and future volatility of ASE. Adaptive Garch model results, the weights of α , B and γ begin with (0.0276), (0.892) and (0.2144) respectively, and ended by (0.00375), (0.7934) and (0.00731) corresponding γ evolution of the parameters are given in figure (2) for Amman stock exchange daily returns during the study period the sum of α , B and γ is always around one, from figure

(2) we can notice that the parameter is shifted up.

Figure (2): Evolution of the Estimated Parameters α ,B and γ for the ASE.



The Garch option price does not seem overly sensitive to (α and B) parameters or the unit risk premium, (λ) variance persistence parameter, $\Omega = \alpha + B$, having magnitude of the equation of Black – Scholes model bias, where the unconditional variance bias is not importantly accurate to justify the model to ASE data. Figure (4) declare the option price difference 2 times.

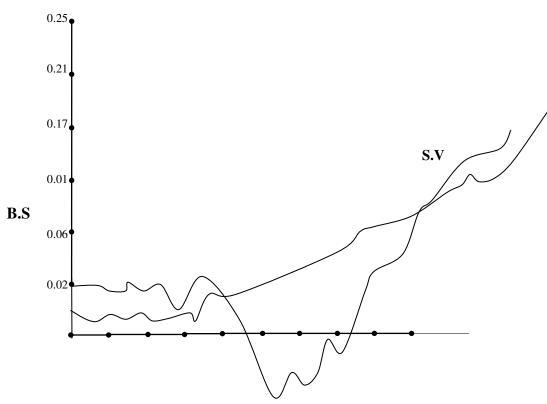
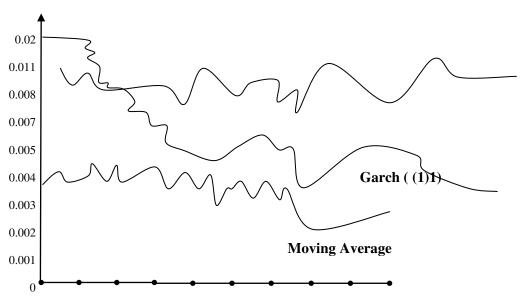
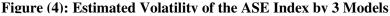


Figure 3: Stock Price of ASE Difference with Volatility of Black – Scholes and Stochastic Volatility

Option prices are different 2 times, in this figure (4): the comparison between the stochastic volatility and the Black – schools, the diagram shows as the difference from point to another this happen due to subtracting the Black –Scholes price of the two models, considered the option price as dummy or default. Also, we can see this volatility, in figure (4).





The Black –Scholes in upper figure is set an option price of ASE to which other models are compared of volatility Figure (4) illustrate the compare between models with a slight volatility, but the volatility become more obvious.

Black - Scholes overprice near and at the money call option of the stock ,while it produces a lower prices for a deep out-of –the money stock price of moving average model , the pattern for implied volatility difference is very small ,the reason behind this may be that the implied model generates a volatility , due to the figure we can conclude that Black - Scholes volatility is less than other models .

Concluded remarks:

This paper uses historical data to estimate the volatility, which give us through analysis a significant bias toward overpricing of the option when we use Black-Scholes equation. Many reasons are behind why we attend to use the model of volatility to be a random process. One of these reasons is it could be simply represent estimation uncertainty, or sometimes it can arise as a fraction of transaction costs, the third reason is it could be, it has a thick (heavy tailed) returns distribution, forth reason has either been as a fair reason is related to any extended model must also specify what type of data can be calibrated with . We can summarize that the calibration procedure as follows: The fit – near the money implied volatilities for several maturities as in ASE ,so straight line in the composite variable called Log – moneyness -to – maturity ratio (LMMR). The estimate of the equation gives us that the slope and the intercept (a) since LMMR= 0 when stock price equal strike price, where b is exactly, that at-the money implied volatility . In addition, we can estimate the historical volatility of stock price returns

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